Stochastic Calculus.

Gautam Iyer, Fall 2013

L1, Mon 8/26.
- Stochastic processes
  - Basic definitions. (Filtrations, stopping times, etc.)
  - Exit times of continuous processes from domains are stopping times.

L2, Wed 8/28.
- Continuous time martingales
  - Doob's Martingale inequalities
  - Existence of RCLL modifications (no proof; read [1, Thm 1.3.13] if interested).
  - Submartingale convergence

L3, Wed 9/4.
  - Optional sampling.
  - Local martingales.

L4, Mon 9/9.
  - Completeness of $\mathcal{M}^2$, $\mathcal{M}_c^2$.
- Quadratic variation.
  - Definition as limit of sums of squares.

L5, Mon 9/10.
  - Proof of existence.
  - Joint quadratic variation.
  - See also [1, §1.4–1.5] for an alternate treatment using the Doob Meyer decomposition.

L6, Wed 9/12.
- Construction of Brownian Motion
  - Construction via consistency and the Kolmogorov Čentsov theorem.

L7, Mon 9/16.
  - Alternate construction, and a direct proof of Hölder continuity.

L8, Mon 9/23.
- Stochastic Integration
  - Construction and Itô isometry.

L9, Wed 9/25.
  - Approximation by simple functions.
  - Joint quadratic variation of Itô integrals.
  - Martingale characterization.

L10, Mon 9/30.
  - Integration with respect to local martingales.
- Itô's formula.
  - Statement.
  - Lévy's criterion (sufficient part).

L11, Wed 10/02.
  - Proof of the Itô formula.
  - The expected exit time of Brownian motion from a domain.
  - Stratonovich integrals.

L12, Wed 10/9.
- Martingale representation theorem
  - An orthogonal decomposition of $\mathcal{M}^2$.

L13, Mon 10/14.
  - Martingale representation theorem for Brownian motion.
  - Itô representation theorem.

L14, Mon 10/21.
- Properties of Brownian Motion
  - Markov and Strong Markov properties.

L15, Wed 10/23.
  - Reflection principle
  - Computation of passage time densities
  - Blumenthal and Kolmogorov 0-1 laws.
  - Zero set of Brownian motion

L16, Mon 10/28.
  - Running maximum
  - Law of iterated logarithm

L17, Wed 10/30.
- The Girsanov Theorem
  - Statement and proof.

L18, Mon 11/4.
  - Passage times of Brownian motion with a drift.
  - Regularity of exponential martingales.

L19, Mon 11/11.
- Stochastic Differential equations
  - Strong solutions, existence and uniqueness.

L20, Wed 11/13.
  - Weak solutions.
    - Tanaka's example.
    - Existence and uniqueness via the Girsanov theorem.

L21, Mon 11/18.
- Diffusions
  - Markov and Strong Markov properties.

L22, Mon 11/25.
  - Dynkin's formula, generators.
  - Recurrence of Brownian Motion.
  - Kolmogorov backward equation.

L23, Wed 11/27.
  - Feynman-Kac formula.
  - Kolmogorov forward equation.

L24, Mon 12/2.
  - Dirichlet-Poisson problems in bounded domains.

References

[1] I. Karatzas and S. E. Shreve, *Brownian motion and stochastic calculus*, 2nd ed., Graduate Texts in Mathematics, vol. 113, Springer-Verlag, New York, 1991. MR1121940 (92h:60127)

[2] B. Øksendal, *Stochastic differential equations*, 6th ed., Universitext, Springer-Verlag, Berlin, 2003. An introduction with applications. MR2001996 (2004e:60102)

[3] H. Kunita, *Stochastic flows and stochastic differential equations*, Cambridge Studies in Advanced Mathematics, vol. 24, Cambridge University Press, Cambridge, 1997. Reprint of the 1990 original. MR1472487 (98e:60096)