720 Measure Theory: Midterm.

Oct 17$^{\text{th}}$

- *This is a closed book test. No calculators or computational aids are allowed.*
- *You have 80 mins. The exam has a total of 4 questions and 20 points.*
- *You may use without proof any result that has been proved in class or on the homework, provided the result you want to use does not rely on the result you have been asked to prove. You must also **CLEARLY** state the result you want to use.*
- *Good luck.*

In this exam, we always assume (X, Σ, μ) is a measure space.

$\boxed{5}$ 1. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function. Must $f' : \mathbb{R} \to \mathbb{R}$ be Borel measurable? Prove, or find a counter example.

$\boxed{5}$ 2. Let $f \in L^1(X)$. Can $\{\alpha \in \mathbb{R} \mid \mu\{f = \alpha\} > 0\}$ be uncountable? Disprove, or find an example.

$\boxed{5}$ 3. Let X be a compact metric space, and μ a finite measure on $(X, \mathcal{B}(X))$. Must μ be regular? Prove, or find a counterexample.

$\boxed{5}$ 4. Does there exist $f : \mathbb{R} \to [0, \infty)$ Borel measurable such that $\int_a^b f \, d\lambda = \infty$ for all $a, b \in \mathbb{R}$ with $a < b \in \mathbb{R}$? Find an example, or disprove the existence.