1. Sec. 4.5. 3, 15
2. Sec. 4.10. 4, 8

3. Let \( D \subseteq \mathbb{R}^2 \) be the region bounded by the lines \( y = x, \ y = x + 2\pi \) and \( x = 0. \)
Let \( f(x, y) = \sin(y - x). \)

(a) Does Fubini’s theorem apply when computing \( \iint_D f(x, y) \, dx \, dy? \)
(b) Verify both the iterated integrals associated with \( \iint_D f(x, y) \, dx \, dy \) are finite, but not equal.

4. Let \( \varphi : \mathbb{R}^2 \to \mathbb{R}^2 \) be \( C^1, \) and suppose for simplicity \( \varphi(0, 0) = (0, 0). \) For \( \varepsilon > 0, \) let \( P_\varepsilon \) be the parallelogram with vertices \((0, 0), \varphi(\varepsilon, 0), \varphi(0, \varepsilon) \) and \( \varphi(\varepsilon, 0) + \varphi(0, \varepsilon). \)
Compute \( \lim_{\varepsilon \to 0^+} \frac{1}{\varepsilon} \text{Area}(P_\varepsilon). \) [The real reason behind the change of variable formula is that \( \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \text{Area}(\varphi(S_\varepsilon)) = |\text{det}(D\varphi)|, \) where \( S_\varepsilon \) is the square with diagonal points \((0, 0)\) and \((\varepsilon, \varepsilon). \) This, however, is a little harder to see, mainly because \( \varphi(S_\varepsilon) \) need not have a “nice shape”. If we approximate \( \varphi(S_\varepsilon) \) by a parallelogram then computing the limit can be done directly, and is exactly the content of this question.]

5. (a) Let \( T : \mathbb{R}^2 \to \mathbb{R}^3 \) be linear, and \( P \) be the parallelogram with sides \( T(e_1) \) and \( T(e_2). \) Find a formula for the area of \( P \) in terms of the matrix of \( T. \)
(b) Let \( U \subseteq \mathbb{R}^2 \) be open, and \( \varphi : U \to \mathbb{R}^3 \) be \( C^1 \) and injective. Then \( S \defeq \varphi(U), \) the image of \( U \) under \( \varphi \) is a surface in \( \mathbb{R}^3. \) Guess a formula for the area of \( S \) in terms of \( \varphi. \) [Hint: Use the same intuition from change of variable: If \( P \) is a really small square in \( U, \) what is the area of \( \varphi(P)? \) The previous subpart helps.]
(c) Let \( f : U \to \mathbb{R} \) be \( C^1. \) Use your formula from the previous subpart to show that
\[
\text{Area}\{(x, y, f(x, y)) \mid (x, y) \in U\} = \iint_U \sqrt{1 + (\partial_x f)^2 + (\partial_y f)^2} \, dx \, dy.
\]
(d) Using your formula, to compute the surface area of a sphere of radius \( r. \)

6. Let \( f : [a, b] \to (0, \infty) \) be differentiable, and \( S \subseteq \mathbb{R}^3 \) be the surface formed by rotating the graph of \( f \) about the \( x \)-axis. Explicitly,
\[
S = \{(x, y, z) \mid x \in [a, b] \text{ and } y^2 + z^2 = f(x)^2\}.
\]
Show that that the area of \( S \) is \( \int_a^b 2\pi f(x) \sqrt{1 + f'(x)^2} \, dx. \) [You might have already seen this formula from one variable calculus. You can derive it here using Fubini’s theorem and the surface area formula from the previous question.]