$\boxed{12.6}$

$d[M,M] = \sigma\, dt$

$d[N,N] = \tau\, dt$

$d[M,N] = \rho\, dt$

σ, τ, ρ Not random.

$\varphi(t) = \mathbb{E}\, e^{\lambda M_t + \mu N_t}$

$d\left(e^{\lambda M_t + \mu N_t}\right) \overset{\text{Ito}}{=} \_\_\_\_\_\_$

take Exp.

$\partial_t \varphi = \_\_\_\_ \, \varphi$ & solve

$\int_0^t W_s \, ds \neq W_s \int_0^t ds$

fn of t

$\int_0^t s \, dW_s \neq s \int_0^t dW_s$

$\boxed{12 \cdot 8}$

$$Z_t = \exp\left(\theta W_t - \frac{\theta^2 t}{2}\right) \leftarrow \quad Z_s = \exp\left(\theta W_s - \frac{\theta^2 s}{2}\right)$$

Find g + $\quad E_s f(Z_t) \overset{\text{Wait}}{=} g(Z_s)$

$$\boxed{Z_t = Z_s \exp\left(\theta(W_t - W_s) - \frac{\theta^2(t-s)}{2}\right)}$$

Compute $\quad E_s f(Z_t) = E_s f\left(\underbrace{Z_s}_{\mathcal{F}_s \text{ meas.}} \exp\left(-\frac{\theta^2}{2}(t-s) + \theta\sqrt{t-s}\,\underbrace{\frac{(W_t - W_s)}{\sqrt{t-s}}}_{\text{Indep of } \mathcal{F}_s}\right)\right)$

$$= \int_{y=-\infty}^{\infty} f\left(Z_s \exp\left(\frac{-\theta^2(t-s)}{2} + \theta\sqrt{t-s}\, y\right)\right) \frac{e^{-y^2/2}}{\sqrt{2\pi}}$$

$$g(x) = \int_{-\infty}^{\infty} f(x \exp(\;)) \frac{e^{-y^2/2}}{\sqrt{2\pi}}\, dy$$

$$\boxed{\begin{array}{l} dX_t = b_t\,dt + 0\,dW_t \\ \Rightarrow X \text{ has finite } 1^{st} \text{ var} \\ \quad \& \quad 0 \text{ QV} \end{array}}$$

W & B indep BM's

$$B_t \int_0^t W_s\,dB_s \underset{\text{NOT OK}}{\cancel{\overset{?}{=}}} \int_0^t \underbrace{B_t W_s}_{\text{Not adapted}}\,dB_s$$

$$B_\tau \int_\tau^t W_s\,dB_s \overset{\text{OK}}{=} \int_\tau^t \underbrace{B_\tau W_s}_{\text{adapted}}\,dB_s$$

$$B_\tau \int_0^t W_s\,dB_s \overset{\text{NOT OK}}{\cancel{=}} \int_0^t \underbrace{B_\tau W_s}_{\text{Not adapted}}\,dB_s$$

$$X_t = \int_0^t \sigma_s\,dW_s \qquad dX_t = \sigma_t\,dW_t \qquad dX_t = \text{uuu}\,(\underbrace{B_t\,dt + 1\,dW}_{d\tilde{W}})$$