4. (b) $\tau = T - t$, $z = y + \gamma_2 \tau$

$$V(\tau, z) = e^{\gamma_1 \tau} C(t, y)$$

" $\partial_z V$ ", " $\partial_z^2 V$ "

$$V(\tau, z) = e^{\gamma_1 \tau} \cdot C(T-\tau, z - \gamma_2 \tau)$$

$$\partial_\tau V = \ldots \partial_t C, \partial_y C, \partial_y^2 C \ldots$$ Chain Rule

$$\partial_z^2 V = \ldots \partial_t C, \partial_y C, \partial_y^2 C \ldots$$

3(c) $I(t,t) = tW_t - \int_0^t s\, dW_s$

$$\mathbb{E}_s[I(t,t)] \stackrel{?}{=} I(s,s)$$

$$\mathbb{E}_s[I(t,t)] = \mathbb{E}_s\left[tW_t - \int_0^t r\, dW_r\right]$$

$$= tW_s - \int_0^s r\, dW_r$$

$$\neq I(s,s).$$

$M_t = \int_0^t r\, dW_r$: mart

1.C $M_t = \int_0^t W_r \, dW_r$

$$\mathbb{E}_s\left[e^{\lambda(M_t - M_s)}\right]$$

$$e^{\lambda(M_t - M_s)} = e^{\lambda \cdot \int_s^t W_r \, dW_r} = e^{\lambda X_t}$$

$$\left(\underline{X_t = \int_s^t W_r \, dW_r}, \quad t \geq s, \quad X_s = 0. \right)$$

$$de^{\lambda X_t} = \lambda \cdot e^{\lambda X_t} dX_t + \frac{\lambda^2}{2} e^{\lambda X_t} d\langle X, X \rangle_t$$

$$\Rightarrow e^{\lambda X_t} - 1$$

$$= \int_s^t \lambda e^{\lambda X_r} \underline{dX_r} + \frac{\lambda^2}{2} \int_s^t e^{\lambda X_r} \underline{d\langle X, X \rangle_r}$$

4(a) $y = \log x$

$$\partial_x C = \frac{dC}{dx} = \frac{\partial C}{\partial y} \cdot \frac{\partial y}{\partial x} = e^{-y}$$ (Chain Rule)