## Continuous Time Finance: Midterm 1.

2023-02-22

- This is a closed book test. You may not use phones, calculators, or other electronic devices.
- You may not give or receive assistance.
- You have 50 minutes. The exam has a total of 5 questions and 40 points.
- The questions are roughly ordered by difficulty. Good luck.

In this exam W always denotes a standard Brownian motion, and the filtration  $\{\mathcal{F}_t \mid t \ge 0\}$  is the Brownian filtration.

- 10 1. Let B be a continuous process with finite first variation. Must [B, B] = 0? If yes, prove it. If no, find a counter example.
- 5 2. Let  $\alpha, s > 0$ . Compute  $E(e^{-\alpha W_s^2})$ . (Your final answer should not involve expectations or integrals.)
- 5 3. If  $0 \le s \le t$ , find  $\mathbf{E} \exp(iW_s(W_t W_s))$ . Express your answer in terms of s and t without using expectations or integrals. (Here  $i = \sqrt{-1}$ .)
- 10 4. Let  $X_t = e^{2tW_t}$ . Find  $[X, X]_t$ . Your answer may involve (unsimplified) Riemann or Ito integrals, but should not involve limits.
- 10 5. Fix T > 0. Compute

$$\lim_{\|P\|\to 0} \sum_{i=0}^{n-1} W_{t_{i+1}} (W_{t_{i+1}} - W_{t_i})$$

Here  $t_0 = 0 < t_1 < \cdots < t_n = T$ ,  $P = \{t_0, \ldots, t_n\}$  is a partition of [0, T], and  $||P|| = \max_i(t_{i+1} - t_i)$  is the mesh size of the partition. Your answer may involve Riemann or Itô integrals, but should not involve limits.