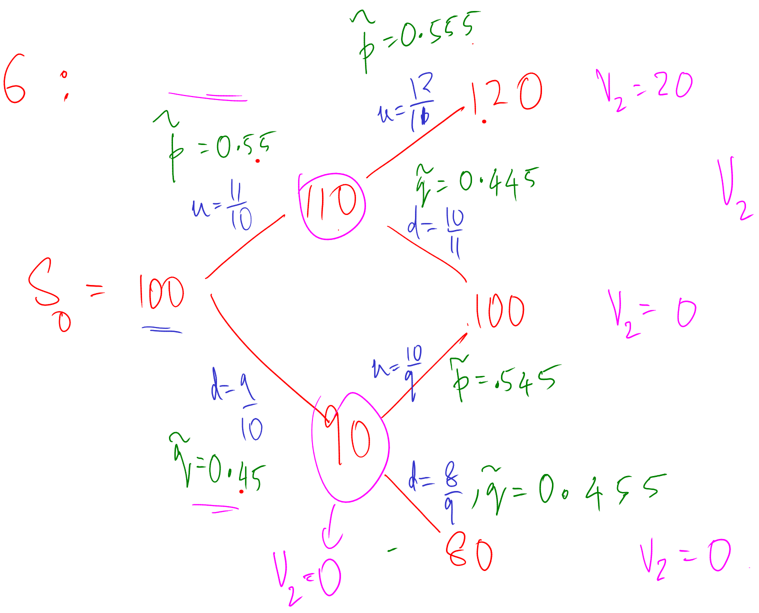
HW 4 Q 6:



Interest rate $r = 0.01$

V_2 = European call strike 100
Maturity 2.

$$V_1(1, *) = \frac{1}{1.01} \tilde{E}_1 V_2$$

$$= \frac{1}{1.01} \left(0.555 (20) + 0.445 (0) \right)$$

$$=$$

$n = 1$

$$\tilde{p} = \frac{1+r-d}{u-d} = \frac{1.01 - 0.9}{0.2} = 0.55$$

$$n=2, \quad w_1 = 1 : \quad \hat{p} = \frac{1+r-d}{u-d} = \frac{1.01 - \frac{10}{11}}{2/11} = 0.555$$

$$n = 2 \quad w_1 = -1 : \quad \hat{p} = \frac{1+r-d}{u-d} = \frac{1.01 - 8/9}{2/9} = 0.545$$

HW 4 Q 2b]

$$X_t = \left(W_t + \frac{t^2}{2}\right) \exp\left(-\int_0^t s\, dW_s - \frac{t^3}{6}\right)$$

Can not choose $f(t,x) = \left(\underbrace{x + \frac{t^2}{2}}\right) \exp\left(-\underline{\underline{\int_0^t s\, dW_s}} - \underline{t^3/6}\right)$

because $\int_0^t s\, dW_s$ is not diff wrt t.

(To apply Ito we need f to have 1 derivative wrt t & 2 derivatives wrt x).

Write $\int_0^t s\, dW_s = (\quad)_W \int_0^t (\quad)\, ds$

Let $f(t,x) = tx$ & Itô:

$t\, dW_t$

$$d(tW_t) = t\, dW_t + W_t\, dt + 0$$

$$\Rightarrow tW_t - 0 = \int_0^t s\, dW_s + \int_0^t W_s\, ds$$

$$\Rightarrow \int_0^t s\,dW_s = tW_t - \underbrace{\int_0^t W_s\,ds}$$

diff wrt t !! $\left(\frac{d}{dt}\int_0^t W_s\,ds = W_t \right)$

$$\Rightarrow X_t = \left(W_t + \frac{t^2}{2}\right) \exp\left(\int_0^t W_s\,ds - tW_t - t^3/6 \right)$$

(Replace W_t with x)

Chose $f(t,x) = \left(x + \frac{t^2}{2}\right)\exp\left(\boxed{\int_0^t W_s\,ds} - tx - \frac{t^3}{6} \right)$

① $\partial_t f = t \exp(\ ) + \left(x + t^2/2\right)\exp(\ ) \cdot \left(W_t - x - \frac{t^2}{2}\right)$

② $\partial_x f = 1 \exp(\ ) + \left(x + t^2/2\right)\exp(\ )\,(-t)$

③ $\partial_x^2 f = \exp(\ )(-t) + \left(1 + \cancel{t^2/2}\right)\exp(\ )(-t) + \left(x + \frac{t^2}{2}\right)(-t)\exp(\ )(-t)$

$$dX = \partial_t f \, dt + \partial_x f \, dW + \frac{1}{2}\partial_x^2 f \, dt$$

$$= \partial_x f \, dW + \exp(\ )\left[t + \left(W_t + \frac{t^2}{2}\right)\left(W_t - W_t - \frac{t^2}{2}\right) + \frac{1}{2}\left(-t - t + t^2\left(W_t + \frac{t^2}{2}\right)\right)\right]dt$$

$$= \partial_x f \, dW + 0$$

Q1 d) $X_t = W_t \int_0^{W_t} \exp(-t s^2) \, ds$

$$f(t,x) = x \int_0^x \exp(-t s^2) \, ds$$

1 diff wrt t

2 & 2 diff wrt x

1) $$\partial_t f = \partial_t \left(x \int_0^x \exp(-t s^2) \, ds \right)$$

$$= x \int_0^x \partial_t \left(e^{-t s^2} \right) ds = \underbrace{x \int_0^x e^{-t s^2} (-s^2)\, ds}$$

$$② \quad \partial_x f = \partial_x \left(x \int_0^x \exp(-t s^2)\, ds \right) = x \exp\left(-\underline{t} x^2\right)$$

$$+ \int_0^x \exp\left(-\underline{t} s^2\right) ds$$

$$dX = \partial_t f \, dt + \partial_x f \, dW + \frac{1}{2} \partial_x^2 f \, dt$$

$$= \left(\partial_t f + \frac{1}{2} \partial_x^2 f \right) dt + \underline{\partial_x f} \, dW$$

$$\left. \right\} \quad X_t = X_0 + \int_0^t b_r \, dr + \int_0^t \underline{\sigma_r} \, dW_r$$

u

Where $\quad \overline{\tau}_r = W_r \exp\left(-r W_r^2\right) + \int_0^{W_r} \exp\left(-r s^2\right) ds$

Q2 c) What do we do with

$$\int_0^t b_s \, dW_s \; ? \longrightarrow (\;\;)_{W_t}$$

Want: $\int_0^t (\;\;) \, ds$

Intuition: Want to Ito something & have a term that looks like

$$\cdot \; \underbrace{b_t}_{} \, dW_t$$

$$f(t,x) = x\, b_t \qquad \partial_t f = b' x \qquad \partial_x f = b \qquad \partial_x^2 f = 0$$

$$d\big(f(t,W_t)\big) = d\big(b_t W_t\big) = b_t' W_t\, dt + b_t\, dW + 0$$

$$\Rightarrow b_t W_t - 0 = \int_0^t b_s' W_s\, ds + \int_0^t b_s\, dW_s$$

$$\Rightarrow \int_0^t b_s\, dW_s = b_t W_t - \int_0^t b_s' W_s\, ds.$$

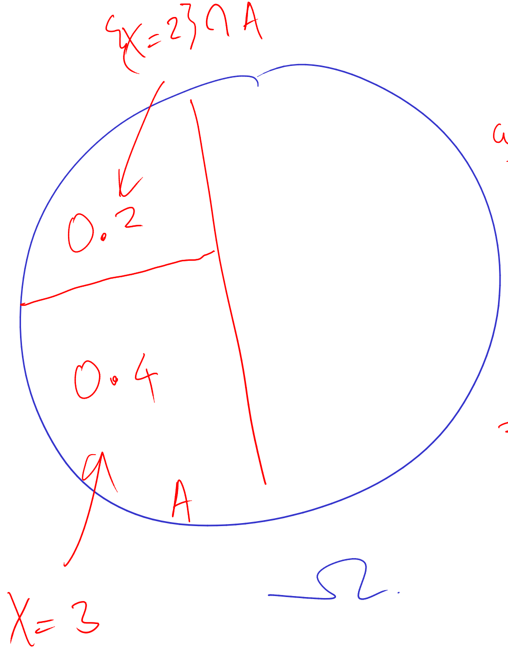
$\boxed{Q:}$

$$\underline{A} = \{E_n X = \alpha\}$$

$$\left.\begin{aligned} P(X=\underline{2}) &= 0.3 \\ P(X=\underline{3}) &= 0.7 \end{aligned}\right\} + = 1$$

$$P(\{X=2\}\cap A) = 0.2$$

$$P(A) = \underline{0.6}$$



$$\sum_{\underline{\omega\in A}} p(\omega)\, \overbrace{E_n X(\omega)}$$

$$= \sum_{\underline{\omega\in A}} p(\omega)\, \underline{X(\omega)}$$

$$\Rightarrow \alpha \overset{0.6}{P(A)} = (0.2)\cdot 2 + (0.4)\cdot 3$$

Answers to 2016 Midterm:

1a X_s

1b $t^2/2$

2 $$X_t = X_0 + \int_0^t e^{3W_s}\left(1 + \frac{9s}{2}\right) ds + \int_0^t 3s\, e^{3W_s}\, dW_s$$

3 $$[Z, Z]_t = \int_0^t \left(s + 2se^{2sW_s}\right)^2 ds$$

4 $$\frac{s^2 W_t^2}{t^2} + \left(1 - \frac{2s}{t}\right)s + \frac{s^2}{t}$$

5 $X_s^2 + (t-s)W_s^2 + \frac{(t-s)^2}{2}$