Continuous Time Finance: Midterm 1.

2022-02-23

- *This is a closed book test. You may not use phones, calculators, or other electronic devices.*
- *You may not give or receive assistance.*
- *You have 50 minutes. The exam has a total of 4 questions and 40 points.*
- *The questions are roughly ordered by difficulty. Good luck.*

In this exam W always denotes a standard Brownian motion, and the filtration $\{\mathcal{F}_t \mid t \geqslant 0\}$ is the Brownian filtration.

10 1. If $0 \leqslant s \leqslant t$, find $\boldsymbol{E}(W_s^2 W_t^2)$. Express your answer in terms of s and t without using expectations or integrals. (You may use the fact that if $X \sim N(0, \sigma^2)$, then $\boldsymbol{E}(X^{2m}) = (\frac{\sigma^2}{2})^m (2m)!/m!$)

10 2. Find $\lim_{\|P\| \to 0} \boldsymbol{E} \sum_{k=0}^{n-1} |W_{t_{k+1}} - W_{t_k}|^{3/2}$ where $P = \{0 = t_0 < t_1 < \cdots < t_n = T\}$ is a partition of $[0, T]$, and $\|P\| = \max_{k \leqslant N}(t_{k+1} - t_k)$ is the mesh size. (Note this is something we did both in class and on the homework. Please provide a complete proof here, without simply quoting the homework/class notes.)

10 3. Let Y be a normally distributed random variable with mean 0 and variance σ^2. Let $K > 0$ and X be a non-negative random variable that is independent of Y. Compute $\boldsymbol{E}((Xe^Y - K)^+ \mid X)$. Express your final answer in terms of σ, X and the cumulative distribution function of the standard normal without involving expectations or integrals.

10 4. Given $0 \leqslant s \leqslant t$, find $\boldsymbol{E}_s\Big[\Big(\int_0^t e^{2r^2} W_r \, dW_r\Big)^2\Big]$. Express your answer in terms of s, t without involving expectations or conditional expectations. (Having an unsimplified Riemann and/or Itô integral in your answer is OK, as long as it doesn't involve expectations or conditional expectations.)