$\boxed{4a}$ $g(x) = E\left(e^{(x+Y)} - K\right)^+$ $\quad Y \sim N(0,1)$

$$= \int_{-\infty}^{\infty} \left(e^{x+y} - K\right)^+ \frac{1}{\sqrt{2\pi}} e^{-y^2/2}\, dy$$

$(K > 0)$

Goal: Simplify & remove the integral

Change limits: Want $e^{x+y} - K \geq 0$

$$\Leftrightarrow e^{x+y} \geq K \Leftrightarrow y \geq \underbrace{\ln K - x}_{c}$$

Let $c = \ln K - x$. $\quad y \geq c \iff e^{x+y} \geq K$

$$\Rightarrow g(x) = \int_{-\infty}^{c} 0 + \int_{c}^{\infty} \left(e^{x+y} - K\right) e^{-y^2/2} \frac{dy}{\sqrt{2\pi}}$$

$$= \underbrace{\int_{c}^{\infty} e^{(x+y) - y^2/2} \frac{dy}{\sqrt{2\pi}}}_{\mathrm{I}} - \underbrace{K \int_{c}^{\infty} e^{-y^2/2} \frac{dy}{\sqrt{2\pi}}}_{\mathrm{II}}$$

$$\mathrm{II}: \int_c^\infty e^{-y^2/2}\,\frac{dy}{\sqrt{2\pi}} = N(-c)$$

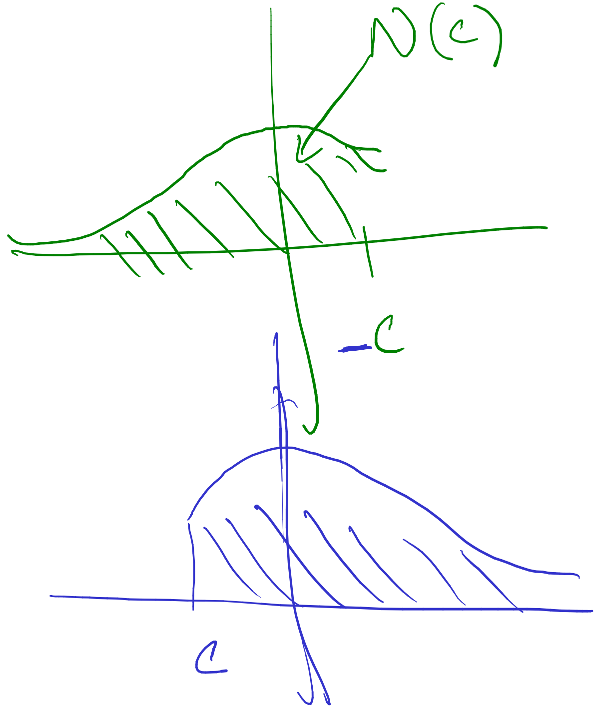
$$\mathrm{I}: \int_c^\infty e^{x+y-y^2/2}\,\frac{dy}{\sqrt{2\pi}}$$

$$= e^x \int_c^\infty e^{-\frac{1}{2}(y^2-2y+1)+1/2}\,\frac{dy}{\sqrt{2\pi}}$$

$$= e^{x+1/2} \int_c^\infty e^{-(y-1)^2/2}\, dy/\sqrt{2\pi}$$

Recall $N(x) = \int_{-\infty}^x e^{-y^2/2}\,\frac{dy}{\sqrt{2\pi}}$

= CDF of normal.



$$= e^{x+1/2} \int_{c-1}^{\infty} e^{-z^2/2} \frac{dz}{\sqrt{2\pi}}$$

$$z = y - 1$$

$$dz = dy$$

$$= e^{x+1/2} N(1-c)$$

$\boxed{\text{HW 3 Q3a}}$ Want $E_s W_t^3$

$$E_s W_t^3 = E_s (W_t - W_s + W_s)^3$$

$$= E_s (W_t - W_s)^3 + 3 E_s\left[(W_t - W_s)^2 W_s\right] + 3 E_s\left[(W_t - W_s) W_s^2\right]$$

$$+ E_s W_s^3$$

$$= E (W_t - W_s)^3 + 3 W_s E_{\cancel{s}} (W_t - W_s)^2 + \text{similarly}$$

$$= 0 + 3 W_s (t - s) + \cdots$$

Q2 $\underline{6}$ Given layer cake: $Y \geq 0$, $EY = \int_0^\infty P(Y \geq t)\, dt$

Want: $\varphi(0) = 0$, φ inc find $E\varphi(X)$ $(X \geq 0)$

L.C. $\Rightarrow E\varphi(X) = \int_0^\infty P(\varphi(X) \geq t)\, dt$

Put $t = \underline{\varphi(s)}$

$dt = \varphi'(s)\, \underline{ds}$

$$\Rightarrow E\varphi(X) = \int_0^\infty P(\varphi(X) \geq t)\, dt = \int_0^\infty P(\underbrace{\varphi(X) \geq \varphi(s)})\, \underline{\varphi'(s)}\, \underline{ds}$$

$$\overset{\text{since } \varphi \text{ is inc}}{=\!=} \int_0^\infty P(X \geq s)\, \varphi'(s)\, ds .$$