

HW2 Q2
 $E X_n \rightarrow \mu_n$

$$E (X_n - \mu_n)^2 = \sigma_n^2$$

Want CLT.

$$(\sigma_n \rightarrow \sigma)$$

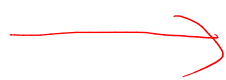
Supposition 1: Say $\mu_n = 0 \quad \forall n.$

Copy CLT statement:

$$S_n = \frac{1}{\sqrt{n}} \sum_1^n X_k$$

Want

$$\frac{S_n}{\sqrt{n}}$$



Normal.

$$\begin{matrix} N(0, \sigma^2) \\ \cancel{N(0, 1)} \end{matrix}$$

COPY the proof!!

Find ϕ_{S_n}

Find φ_{X_n} first

guess $\varphi_{X_n}(\lambda) = 1 + \underbrace{0 \lambda}_{\text{circled}} - \frac{\sigma^2}{n} \lambda^2 + \dots$

$$\varphi_{S_n}(\lambda) = E e^{i\lambda S_n} = \prod_{k=1}^n \left(1 - \frac{\sigma_k^2}{2} \lambda^2 + \dots \right)$$

$$\varphi_{\frac{S_n}{\sqrt{n}}}(\lambda) = \varphi_{S_n}\left(\frac{\lambda}{\sqrt{n}}\right) = \prod_{k=1}^n \left(1 - \frac{\sigma_k^2}{2n} \lambda^2 + \underbrace{O\left(\frac{\lambda^3}{n^{3/2}}\right)}_{\text{ignore}} \right)$$

$$\mathbb{Q}_0^0 \quad \sigma_k \xrightarrow{k \rightarrow \infty} \sigma$$

find $\lim_{n \rightarrow \infty} \prod_{k=1}^n \left(1 - \frac{\sigma_k^2 \lambda^2}{2n} + O\left(\frac{\lambda^3}{n^{3/2}}\right) \right) = L.$

$$\Rightarrow \underline{\ln L} = \lim_{n \rightarrow \infty} \sum_{k=1}^n \ln \left(1 - \frac{\sigma_k^2 \lambda^2}{2n} + O\left(\frac{\lambda^3}{n^{3/2}}\right) \right)$$

$$\ln(1+x) \approx x$$

$$= \lim_{n \rightarrow \infty} \underbrace{\sum_{k=1}^n \left(-\frac{\sigma_k^2 \lambda^2}{2n} + O\left(\frac{\lambda^3}{n^{3/2}}\right) \right)}_{\text{bracketed}}$$



$$\begin{aligned} Q_0: \lim_{n \rightarrow \infty} \frac{-\lambda^2}{2^n} \sum_{k=1}^n \sqrt{\sigma_k^2} &= -\frac{\lambda^2}{2} \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \sqrt{\sigma_k^2} \\ &= -\frac{\lambda^2}{2} \left(\sqrt{\sigma^2} \right) \end{aligned}$$

$$Q_0: \lim_{n \rightarrow \infty} \sum_{k=1}^n O\left(\frac{\lambda^3}{n^{3/2}}\right) \leq \underline{=} O\left(\frac{\lambda^3}{n^{3/2}}\right) \leq \frac{C}{n^{1/2}} \rightarrow 0$$

$$\text{Guess } \frac{S_n}{\sqrt{n}} \rightarrow N(0, \sigma^2)$$

(in the case that $\mu_n = 0 \quad \forall n$)

Q3:

Compute $E W_s^2$, $E W_t^2$, $E W_s W_t$

Know

- (a)
- (b)

$$\left. \begin{array}{l} W_t - W_s \sim N(0, t-s) \\ W_t - W_s \text{ ind of } \mathcal{F}_s \end{array} \right\}$$

Put $s=0$
 $W_0 = 0$

$\Rightarrow W_t - W_0 \sim N(0, t-0)$

$\Rightarrow W_t \sim N(0, t)$