$S_0 u^{n_0} = U$

S_0

$S_0 d^{n_0} = L$

Q1: find f so that $f(S_{\tau \wedge n})$ is a mg

Guess: $f(x) = \underset{\sim}{A} + \underset{\sim}{B}\, x^{\alpha}$

$$\tilde{E}_n f(S_{n+1}) = f(S_n)$$

$$\iff \tilde{p}\, f(us) + \tilde{q}\, f(ds) = f(s)$$

& solve for α.

$$\Longleftrightarrow \tilde{p}\, u^\alpha S^\alpha + \tilde{q}\, d^\alpha S^\alpha = S^\alpha$$

$$\Longleftrightarrow \tilde{p}\, u^\alpha + \tilde{q}\, d^\alpha = 1 \qquad \text{\& solve}$$

$$\tilde{p}\, u^\alpha + \frac{\tilde{q}}{u^\alpha} = 1 \qquad \text{\& use quadratic formula.}$$

HW 14 Q 2: AFP of option that pays at time τ $\begin{cases} A & \text{if } S_\tau = L \\ B & \text{if } S_\tau = U \end{cases}$

Expect AFP at time n = $\frac{1}{D_n} \tilde{E}_n(D_\tau V_\tau)$

AFP at time 0 $= \tilde{E}(D_T V_T) = \tilde{E}\left(\frac{V_T}{(1+r)^T}\right)$

find a, b, α + Q $\frac{a + b S_u^{\alpha}}{(1+r)^u}$ is a mg

Find a fn $f(x) = \underline{a + b x^{\alpha}}$

$f(L) = A$

$f(U) = B$

& $\underline{D_n f(S_n)}$ is a $\tilde{P}$ mg

Once we have f, then $V_0 = \tilde{E}(D_\tau V_\tau) = \tilde{E}\big(\overbrace{D_\tau f(S_\tau)}\big)$

$\overset{OST}{=} D_0 f(S_0) = \underline{f(S_0)}$ ← Ans.

Let's find f: $\tilde{E}_n \frac{1}{(1+r)^{n+1}} f(S_{n+1}) \overset{\text{Want}}{=} \frac{1}{(1+r)^n} f(S_n)$

$\iff \frac{1}{1+r} \tilde{E}_n f(S_{n+1}) = f(S_n)$

$$\Longleftrightarrow \quad \frac{1}{1+r}\left(\tilde{p}\, f(u S_n) + \tilde{q}\, f(d S_n)\right) = f(S_n)$$

$$\Longleftrightarrow \quad \frac{1}{1+r}\left(\tilde{p}\, f(u s) + \tilde{q}\, f(d s)\right) = f(s) \qquad \forall\, s \in \text{Range}(S_n)$$

Guess $f(s) = s^{\alpha}$ & solve:

$$\Longrightarrow \quad \tilde{p}\, u^{\alpha} + \frac{\tilde{q}}{u^{\alpha}} = (1+r) \qquad \text{\& find } \alpha$$

Get 2 solutions: $\alpha = 1$ & $\alpha = \alpha_0$

$\Rightarrow$ Choosing $f(s) = a\,s + b\,s^{\alpha_0}$

will ensure $D_n f(S_n)$ is a $\tilde{P}$ mg

Solve for a & b so that $a\,s + b\,s^{\alpha_0} = \begin{cases} A & \text{when } s = L \\ B & \text{when } s = U \end{cases}$

Then $a S_0 + b\, S_0^{\alpha_0} = V_0 = AFP$!

Q: Find $P(S_\tau = L)$:

↳ Method: Know $f(S_n)$ is a mg

provided $f(x) = a + b x^\alpha$ & α is from HW8 Q4

Choose a & b so that $f(x) = a + b x^\alpha = \begin{cases} 0 & x = U \\ 1 & x = L. \end{cases}$

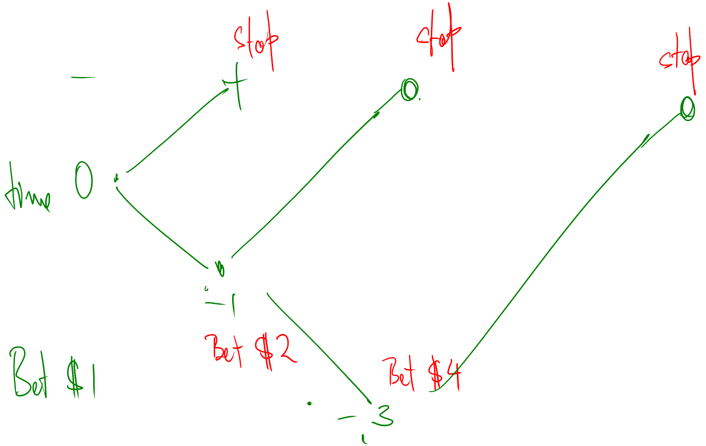
$$\Rightarrow a + b S_0 = f(S_0) = E f(S_\tau) = P(S_\tau = L) \cdot \underbrace{f(L)}_{1} + P(S_\tau = U) \underbrace{f(U)}_{0}$$

HW 14 Q1

$X_n = \pm 1$ (coin flips, iid)

Bet B_n at time n

Reward $B_n X_{n+1}$ at time $n+1$



Bet $\$$ e

-7

M_n = winnings up to time n.

$$M_{n+1} = \underline{\underline{M_n}} + \underline{\underline{B_n}} X_{n+1}.$$

Mg?

$$E_n(M_{n+1}) = M_n + E_n(B_n X_{n+1})$$
$$= M_n$$

$$\mathbb{E}\tau = \sum \underbrace{P(\tau \geq n)}_{\frac{1}{2^n}}$$