3a

$$\begin{pmatrix} S^0_{n+1}(\omega', 1) \\ \vdots \\ S^0_{n+1}(\omega', M) \end{pmatrix}$$

$$\left.\begin{array}{l} \Delta^0_n = 1 \\ \Delta^i_n = 0 \quad i > 0 \end{array}\right\}$$

~~Be~~ At time n,

$\left.\begin{array}{l} \text{Buy } \underline{1} \text{ share of } S^1 \\ \text{Borrow from bank} \end{array}\right\}$

$$\Delta_n = \begin{pmatrix} -S^1_n / S^0_n \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \Delta_n \cdot S_n = 0$$

Let $\Delta_{n,i}$ = trading strat when you buy 1 share of i^{th} stock & borrow from bank

$$\Delta_{n,i} \cdot S_{n+1} = \begin{pmatrix} \Delta_{n,i} \cdot S_{n+1}(\omega', 1) \\ \vdots \\ \Delta_{n,i} \cdot S_{n+1}(\omega', M) \end{pmatrix} \in V$$

$$\left(V = \operatorname{span}\{ \Delta_{n,i} \cdot S_{n+1} \} \right)$$

Binom.

$$\boxed{\tilde{X}_{n+1} = \tilde{\Delta}_n \tilde{S}_{n+1} + (1+r)\left(\tilde{X}_n - \tilde{\Delta}_n \tilde{S}_n\right)}$$ $(**)$

Start with $(\tilde{X}_0, \tilde{\Delta}_0)$

Get $\tilde{X}_1$. Choose $\tilde{\Delta}_1$ arb.

Get $\tilde{X}_2$ etc.

Multi asset:

$$\Delta_n \begin{cases} \Delta_n^0 \longrightarrow \# \text{ shares of MM } S^0 \\ \Delta_n^1 \longrightarrow \# \text{ shares of } S^1 \end{cases}$$

Can choose Δ_{n+1} arb, provided

$$\boxed{\Delta_n \cdot S_{n+1} = \Delta_{n+1} \cdot S_{n+1}} \quad (*)$$

Binom

At time n $\begin{cases} \tilde{\Delta}_n \text{ shares of } \tilde{S}_1 \\ \underbrace{\tilde{X}_n - \tilde{\Delta}_n \tilde{S}_n}_{} \text{ cash} \end{cases}$

i.e.

$$\underline{\underline{\Delta_n}} = \begin{pmatrix} (\tilde{X}_n - \tilde{\Delta}_n S^1_n)/S^0_n \\ \tilde{\Delta}_n \end{pmatrix}$$

Given $\tilde{X}_{n+1} = \tilde{\Delta}_n \tilde{S}_{n+1} + (1+r)(\tilde{X}_n - \tilde{\Delta}_n \tilde{S}_n)$

WTS $\Delta_n \cdot S_{n+1} = \Delta_{n+1} \cdot S_{n+1}$

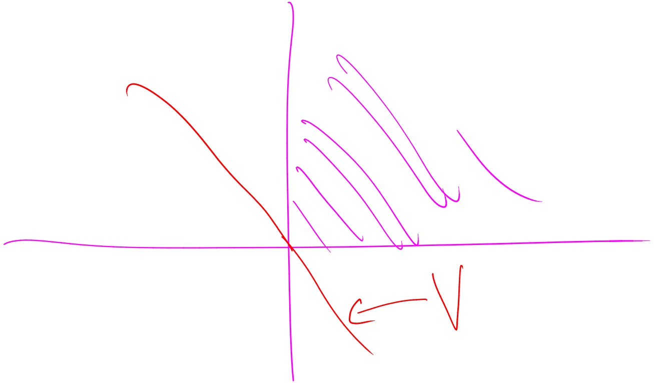
Check: (1) $\Delta_n \cdot S_{n+1} = \begin{pmatrix} (\tilde{X}_n - \tilde{\Delta}_n S^1_n)/S^0_n \\ \tilde{\Delta}_n \end{pmatrix} \cdot \begin{pmatrix} S^0_{n+1} \\ S^1_{n+1} \end{pmatrix}$

Plug in & check (*) & (**) are equiv.!

$$= (1+r)\left(\tilde{X}_n - \tilde{\Delta}_n S'_n\right) + \tilde{\Delta}_n S'_{n+1}$$

② $$\Delta_{n+1} \cdot S_{n+1} = \left(\tilde{X}_{n+1} - \tilde{\Delta}_{n+1} S'_{n+1}\right) + \tilde{\Delta}_{n+1} S'_{n+1}$$

equal.



HW 12 Q2

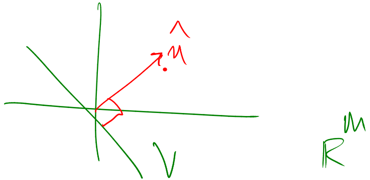
Given $\dim(V) = M-1$ & $V \cap \overline{Q} = \{0\}$

NTS $\exists!$ unit normal $\hat{n} \in \mathring{Q}$

① Know $\exists$ two unit normal $\hat{n} \in \mathbb{R}^M$

② Know if $v \in \mathbb{R}^M$ & $v \cdot \hat{n} = 0 \Rightarrow v \in V$

(uses the fact that $\dim(V) = M-1$)



(3) $\hat{u}_i \neq 0 \quad \forall i$ $\quad \left(\because \text{ Say } \hat{u}_1 = 0 \Rightarrow e_1 \cdot \hat{u} = 0 \right.$

$\left. \Rightarrow e_1 \in V \cap \bar{Q} \text{ contradiction} \right)$

(4) ~~Re~~ Replace $\hat{u}$ by $-\hat{u}$ if necessary

& assume $\hat{u}_1 > 0$

WTS $\hat{u}_2 > 0$: Say $\hat{u}_2 \leq 0$.

$$\begin{pmatrix} \hat{u}_1 \\ \hat{u}_2 \\ * \\ * \\ * \end{pmatrix} \cdot \begin{pmatrix} -\hat{u}_2 \\ u_1 \\ \\ \\ \end{pmatrix} = 0$$

$$\Rightarrow \begin{pmatrix} -\hat{u}_2 \\ u_1 \\ 0 \\ 0 \end{pmatrix} \in V \cap \bar{Q} \quad \text{contradiction!}$$