Binom: $\Delta_0^1 \in \mathbb{R}$

$\Delta_0^1 = \#$ shares held at time 0

Ⓔ Self fin: $X_{n+1}^1 = \Delta_n^1 S_{n+1} + (X_n^1 - \Delta_n^1 S_n^1)(1+r)$

Multiasset notation

$\Delta_0 = (\Delta_0^0, \Delta_0^1)$

$\Delta_0^1 = \Delta_0^1$

$\Delta_0^0 = \text{cash}$

Self fin $\Delta_n \cdot S_{n+1} = \Delta_{n+1} \cdot S_{n+1}$

Δ_0^1 shares of stock

$X_0^1 - \Delta_0 S_0$ in bank

time 1: Δ_1^1 shares of stock

$X_1 - \Delta_1 S_1$ in bank.

$$\Delta_0^1 = \Delta_0^1$$

$$\Delta_0^0 = X_0^1 - \Delta_0 S_0$$

$$\Delta_1^1 = \Delta_1^1$$

$$\Delta_1^0 = \frac{X_1 - \Delta_1 S_1^1}{1+r}$$

$$\boxed{\text{Check } \Delta_0 \cdot S_1 = \Delta_1 \cdot S_1}$$

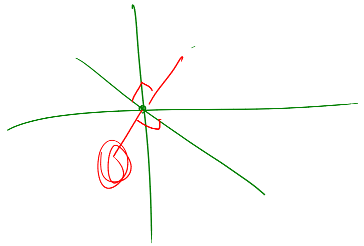
$$v = \begin{pmatrix} v_1 \\ \vdots \\ v_M \end{pmatrix} \qquad \hat{n} = \begin{pmatrix} \hat{n}_1 \\ \vdots \\ \hat{n}_M \end{pmatrix} \qquad v \cdot \hat{n} = v_1 \hat{n}_1 + v_2 \hat{n}_2 \cdots v_M \hat{n}_M$$

$$v \in \mathbb{R}^M, \qquad N = \{ n \in \mathbb{R}^M \mid n \perp V \}$$

$$(\text{i.e. } n \cdot v = 0 \ \forall v \in V.)$$

<u>Claim</u> N is a subspace & $\underbrace{\dim(V)}_{M-1} + \dim(N) = M$

$$\Rightarrow \dim(W) = 1$$



Say $\dim(V) = M-1$, $\quad V \cap \bar{Q} = \{0\}$,

$\hat{n} \perp V$, $\quad |\hat{n}| = 1$, $\quad \hat{n}_1 > 0$

NTS $\hat{n}_i > 0 \quad \forall i$

Say $\hat{n}_2 \leq 0 \qquad \hat{n} = \begin{pmatrix} >0 \\ \leq 0 \\ \times \\ \times \\ \times \end{pmatrix}$

$$\hat{u} = \begin{pmatrix} 1 \\ -1 \\ \cancel{0} \\ \vdots \\ \cancel{0} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = 0$$

found $v \in \mathbb{R}^n$ $\to$ $\hat{u} \cdot v = 0$

Must $v \in V$?

(Yes because $\dim(V) = n-1$)

$$\Rightarrow \begin{pmatrix} 1 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \in V$$

$$\Rightarrow V \cap \overline{Q} \neq \{0\}$$