HW 1 Q1 d

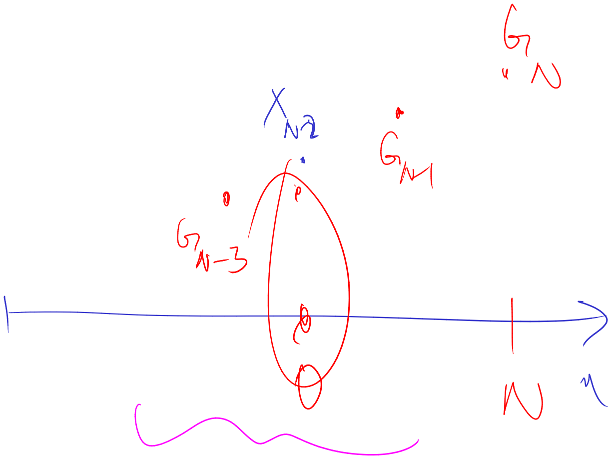
Assume $V_{\tau^*} = G_{\tau^*}$

(NTS τ^* not optimal

(by example.

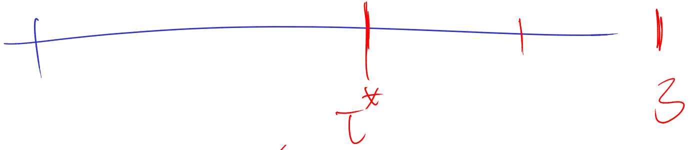
Choose max

$V_n = \left\{ \frac{1}{D_n} E_{n+1} V_{n+1},\ G_n \right\}$



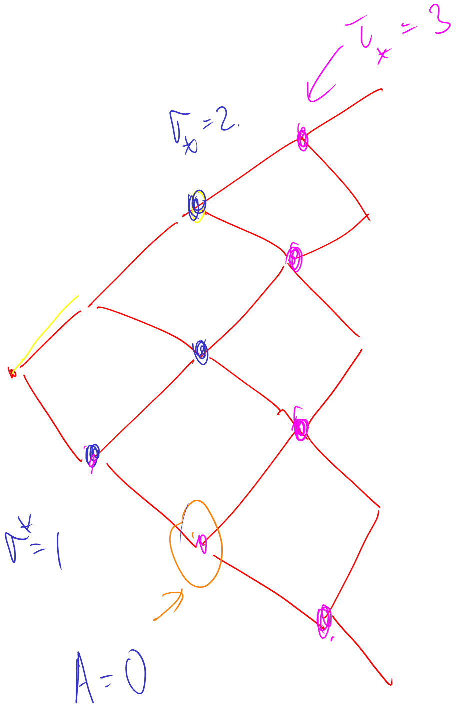
$A_{\tau^*} = 0$ but $V_{\tau^*} > G_{\tau^*}$ $\Rightarrow$ not optimal

Q1 c



$$\tau^* = \max\{n \mid A_n = 0\}$$

Any time $\sigma > \tau^*$ is not optimal.



$$\boxed{A_{\sigma^*} = A_{\tau^*} = 0}$$

$\forall$ stopping times σ, $\sigma^* \le \sigma \le \tau^*$ $\Rightarrow A_\sigma = 0$

[G Hint at C- eq for 1 C]

$N = 3$, $r = 0$, find G so that $\boxed{A_1 > 0}$

Change G_2 so that $G_2 = V_2$

$V \to$ Have formula.

$$D_n V_n = \underbrace{D_n X_n}_{Mg} - \underbrace{A_n}_{Pred}$$

$$\left.\begin{aligned} E_n(D_{n+1} V_{n+1}) &= E_n D_{n+1} X_{n+1} - A_{n+1} \\ & D_n X_n \end{aligned}\right\} \Rightarrow A_{n+1} = A_n + E_n(D_{n+1} V_{n+1}) - D_n V_n$$

3a $\quad G_n = (K - S_n)^+$

$$V_n = \max\left\{G_n, \frac{1}{D_n}\tilde{E}_{n+1}(V_{n+1})\right\}$$

say $V_{n+1} = f(S_{n+1})$ $\leftarrow$ $\max_{b_{n+1}}(S_{n+1})$

$$V_n = \max\left\{(K-S_n)^+, \frac{1}{1+r}\left(f(uS_n)\tilde{p} + f(dS_n)\tilde{q}\right)\right\}$$

$V_n = f(S_n)$, $\boxed{f(s) = \max\left\{ (K-s)^+ ,\ \frac{1}{1+r}\left(f(us)\tilde{p} + f(ds)\tilde{q} \right) \right\}}$

Q3c from 3b

$$f(s) = \begin{cases} 4-s & s \le 2 \\ \frac{4}{s} & s \geq 2 \end{cases} \leftarrow$$

$r = \frac{1}{4}$

disc wealth mg $\rightarrow$ can hold

disc wealth super mg $\rightarrow$ should exercise

$$\boxed{\tilde{p} = \tilde{q} = \frac{1}{2}}$$

$X_n = f(S_n)$ When is $D_n X_n$ a mg / super mg?

Say $S_n > 2$: $\frac{1}{1+r} E_n X_{n+1} = \frac{4}{5} E_n \left(\frac{4}{S_{n+1}} \right)$

$(\Rightarrow S_{n+1} \geq 2)$

$$= \frac{4}{5} \left(\frac{1}{2} \frac{4}{2 S_n} + \frac{1}{2} \frac{4}{S_n/2} \right)$$

$$= \frac{4}{S_n} = f(S_n) = X_n. \text{ mg}$$

Guess: optimal exercise policy $\longrightarrow$ exercise as soon as $S_n \leq 2$.