Q HW 9 Q 5 6 American opt

Rep part 1

$$X_n \geq G_n$$

$$X_{\sigma^*} = G_{\sigma^*}$$

Rep part 2

$$Y_n \geq G_n$$

$$Y_{\tau^*} = G_{\tau^*}$$

Claim: $X_{\sigma^*} = Y_{\sigma^*}$ & $X_{\tau^*} = Y_{\tau^*}$

(1) Check $X_{\tau^*} = Y_{\tau^*}$

(a) Know $D_n X_n$ & $D_n Y_n$ are $\tilde{P}$ mg

(b) $X_{\tau^*} = G_{\tau^*} \le Y_{\tau^*}$

$$\Rightarrow D_{\tau^*} X_{\tau^*} = D_{\tau^*} G_{\tau^*} \le D_{\tau^*} Y_{\tau^*}$$

(c) $\tilde{E}(D_{\tau^*} X_{\tau^*}) = \tilde{E}(D_{\tau^*} G_{\tau^*}) \le \tilde{E}(D_{\tau^*} Y_{\tau^*})$

(d) OST $\Rightarrow$ $\forall$ two bdd stopping time σ, τ

& any mg M, $\quad E M_\sigma = E M_\tau$

(e) $\tilde{E}(D_{\sigma^*} X_{\sigma^*}) \overset{OST}{=} \tilde{E}(D_{\tau^*} X_{\tau^*}) \geq \tilde{E}(D_{\tau^*} G_{\tau^*}) = \tilde{E}(D_{\tau^*} Y_{\tau^*})$

$$\overset{OST}{=} \tilde{E}(D_{\sigma^*} Y_{\sigma^*})$$

(f) $\Rightarrow \tilde{E}(D_{\sigma^*} X_{\sigma^*}) = \tilde{E}(D_{\sigma^*} G_{\sigma^*}) = \tilde{E}(D_{\sigma^*} Y_{\sigma^*})$

g $\Rightarrow$ $D_{\sigma^*} X_{\sigma^*} = D_{\sigma^*} G_{\sigma^*} = D_{\sigma^*} Y_{\sigma^*}$ $\left(\because D_{\sigma^*} Y_{\sigma^*} \geq D_{\sigma^*} G_{\sigma^*} \right)$

$$\boxed{\Rightarrow \quad X_{\sigma^*} = Y_{\sigma^*}}$$

h Same logic $\Rightarrow$ $X_{\tau^*} = Y_{\tau^*}$ $\Rightarrow$ Claim!

i $\Rightarrow$ $\mathbb{1}_{\{n \leq \sigma^*\}} X_n \overset{\text{Wait}}{=} \mathbb{1}_{\{n \leq \sigma^*\}} Y_n$

$$\mathbb{1}_{\{n \le \sigma^*\}} D_n X_n \overset{\text{OST}}{=} \mathbb{1}_{\{n \le \sigma^*\}} \tilde{E}_n\left(D_{\sigma^*} X_{\sigma^*}\right)$$

$$= \mathbb{1}_{\{n \le \sigma^*\}} \tilde{E}_n\left(D_{\sigma^*} Y_{\sigma^*}\right)$$

$$\overset{\text{OST}}{=} \mathbb{1}_{\{n \le \sigma^*\}} D_n Y_n$$

HW 1 Q 6 d]

(a) $D_n X_n$ is a $\tilde{P}$ - mg

(b) $X_n \geq G_n$

Rep Port of american opt.

$\boxed{\text{self fin}}$

(1) $X_n \rightarrow$ self fin

$\Leftrightarrow$ $\boxed{D_n X_n \text{ is a } \tilde{P} \text{ mg}}$

(2) $X_n \geq G_n$ & $\underbrace{X_{\tau^*} = G_{\tau^*}}_{\text{for some stopping time } \tau^*}$