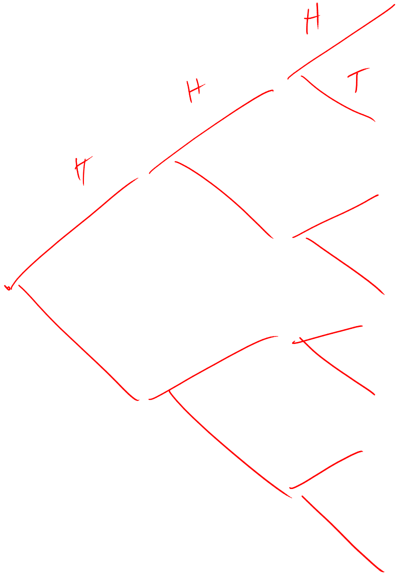
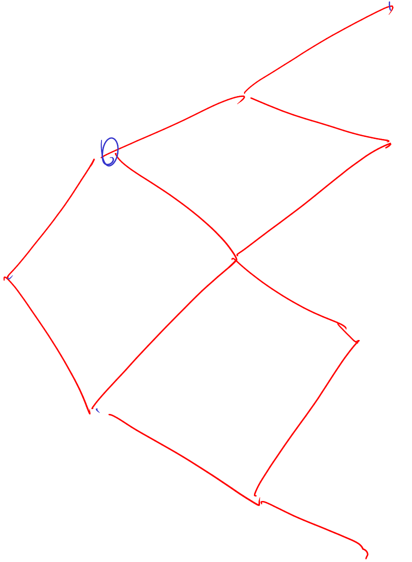
(all coin tosses)



Tree you need to draw to

Stock prices:



Tree you need to draw

Compute $\frac{1}{D_n} E_n V_N D_N$ for state processes

(2^N times)

Check $\tau = 5$ is a stopping time.

NTS $\{\tau = n\} \in \mathcal{F}_n \quad \forall\ \underline{\underline{n}} \in \underline{\{0, 1, \dots N\}}$

Note $\{\tau = n\} = \begin{cases} \underline{\underline{\Omega}} & n = 5 \\ \phi & n \neq 5 \end{cases}$

Both $\Omega \in \mathcal{F}_n$ & $\phi \in \mathcal{F}_n \quad \forall n \Rightarrow \tau$ is a stopping time.

Prop 6.37: Given σ, τ are stopping times.

NTS $\sigma \wedge \tau$ is also a stopping time.

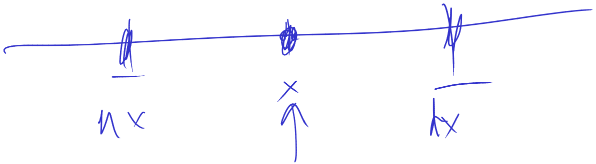
NTS $\forall n$, $\{\sigma \wedge \tau = n\} \in \mathcal{F}_n$.

Note: $$\{\sigma \wedge \tau = n\} = \left(\underset{\in \mathcal{F}_n}{\{\sigma = n\}} \cap \underset{\in \mathcal{F}_n}{\{\tau \geq n\}} \right) \cup \left(\underset{\in \mathcal{F}_n}{\{\tau = n\}} \cap \underset{\in \mathcal{F}_n}{\{\sigma \geq n\}} \right)$$

(Note $\{\tau \geq n\} = \{\tau < n\}^c \in \mathcal{F}_n$)

Finite diff eq:

$$f(x) = c_1 \underline{f(ux)} + c_2 \underline{f(dx)}$$



Up rebate: paid A the first time $S_n \geq U$

State process: $Y = S$.

$$\mathbb{1}_{\{\sigma = n\}} V_n = A \qquad (n < N)$$

$$\mathbb{1}_{\{\sigma = N\}} V_N = \begin{cases} A & S_N \geq U \\ 0 & S_N < U. \end{cases}$$

Let $f_N(s) = A \, \mathbb{1}_{\{s \geq U\}}$.

$$f_n(s) = \mathbb{1}_{\{s \geq U\}} A + \mathbb{1}_{\{s < U\}} \frac{1}{1+r}\left(\tilde{p} f_{n+1}(us) + \tilde{q} f_{n+1}(ds)\right)$$

Price an American ~~put~~ option. as a state process.

Intrinsic value : $g_n = |S_n - K|$

1. Start with $f_N(s) = |s-K|$

(2) $$f_n(s) = \max\left\{ |s-K|,\ \frac{1}{1+r}\left(\tilde{p}\, f_{n+1}(us) + \tilde{q}\, f_{n+1}(ds)\right) \right\}$$