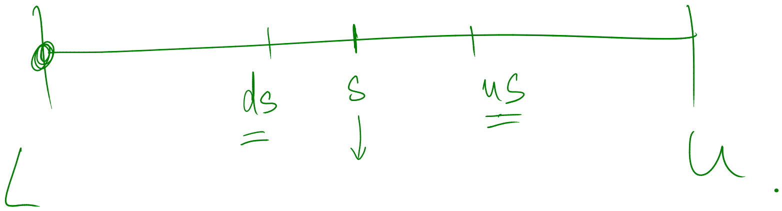
HW 8 Q4 a)

$$f(s) = \mathbb{1}_{\{s \in (L,U)\}} \left(p f(su) + q f(ds) \right) + \mathbb{1}_{\{s \in \{L,U\}\}} f(s)$$



4G Q: What functions satisfy

$$f(s) = p\,f(us) + q\,f(ds) \quad \forall s \quad \ldots (*)$$

→ Eg: $f(s) = 0 \ \forall s$, satisfies (*)

Eg 2: $f(s) = A \ \forall s$ satisfies (*)

Try $f(s) = s^{\alpha}$: Want $s^{\alpha} = p(us)^{\alpha} + q(ds)^{\alpha}$

$$\Leftrightarrow \quad 1 = p\,u^{\alpha} + q\,d^{\alpha}$$

Solve for α: (know $u = \frac{1}{d}$)

$$1 = p\,u^{\alpha} + q\,u^{-\alpha}$$

$$\Leftrightarrow \quad 0 = p\,u^{2\alpha} - u^{\alpha} + q$$

$$\Leftrightarrow \quad u^{\alpha} = \frac{1}{2p}\left(1 \pm \sqrt{1 - 4pq}\right)$$

Have $\cancel{X}$ solutions to $\circledast$: $f(s) = \underline{A} + \underline{B}\, s^{\alpha}$ & α solves !

$$\left.\begin{aligned} f(L) = 0 &= A + BL^{\alpha} \\ f(U) = 1 &= A + BU^{\alpha} \end{aligned}\right\} \begin{array}{l} \alpha \text{ is known,} \\ \text{linear eq in } A \ \& \ B. \\ \text{Solve for } A \ \& \ B \end{array}$$

Note: If $p = q = \frac{1}{2}$, α: $u^{\alpha} = \frac{1}{1}\left(1 \pm \sqrt{1 - 4\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}\right)$

$= 1 \quad \Rightarrow \alpha = 0$

Q3] Opt. State process : $\boxed{Y_{n+1} = h_{n+1}(Y_n, \omega_{n+1}).}$

$\tau = \min\{k \mid Y_k \in A_k\}$

Payoff : $G_\tau = g_\tau(Y_\tau)$ ($g_0 \ldots g_N$ given fns).

Want $\mathbb{1}_{\{n \le \tau\}} V_n = \mathbb{1}_{\{n \le \tau\}} f_n(Y_n)$ & we $\boxed{\text{need for } f_n}$.

Known

$$\underbrace{\mathbb{1}_{\{\tau_n \le \tau\}} V_n}_{} = \underbrace{\frac{1}{1+r}\tilde{E}\, V_{n+1}\, \mathbb{1}_{\{n \le \tau\}}}$$

$$= \mathbb{1}_{\{\tau = n\}}\, G_\tau + \mathbb{1}_{\{\tau > n\}} \frac{1}{1+r} \tilde{E}_n \underline{V_{n+1}}$$

(Ind)

$$= \underbrace{\mathbb{1}_{\{\tau = n\}}}_{\mathbb{1}_{\{Y_n \in A_n\}}} \underbrace{f_n(Y_n)}_{} + \underbrace{\mathbb{1}_{\{\tau > n\}}}_{\mathbb{1}_{\{Y_n \notin A_n\}}} \frac{1}{1+r} \tilde{E}_n \underbrace{f_{n+1}(Y_{n+1})}_{f_{n+1}(h_{n+1}(Y_n,\, \omega_{n+1}))}$$

When $\{n \le \tau\}$

know $\{\tau = n\} = \{Y_n \in A_n\}$

& $\{\tau > n\} = \{Y_n \notin A_n\}$

Indep lemma

get some fn of $\underline{\underline{Y_n}}$

Q.1: Stupid code:

$\boxed{f[n][s] = \text{AFP}_n \text{ when stock price} = s \text{ at time } n}$

Rec rel :

$$f[n][s] = \frac{\tilde{p}\, f[n+1][us] + \tilde{q}\, f[n+1][ds]}{1+r}$$

Better code :

$f[n][k]$ = AFP at time n when stock price $= S_0 u^k d^{n-k}$

Rec rel :

$$f[n][k] = \frac{\tilde{p}\, f[n+1][k+1] + \tilde{q}\, f[n+1][k]}{1+r}$$