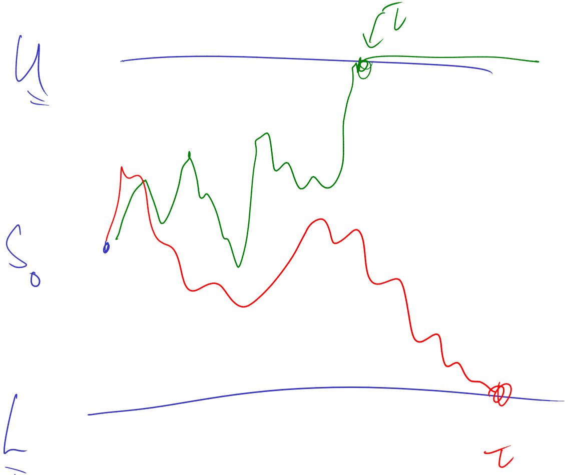
$u^n \circ S_0 = \underline{U}$



$d^m \circ S_0 = \underline{L}$

Want $\underline{f}(S_{\tau\wedge n})$ to be a mg.

Want $f(S_{\tau\wedge n}) = \underbrace{E_n f(S_{\tau\wedge (n+1)})}_{\text{try \& compute}}$

$$E_n\left[\mathbb{1}_{\{\tau \geq n+1\}} f(S_{n+1}) + \mathbb{1}_{\{\tau < n+1\}} f(S_\tau)\right]$$

$$= E_n\left(\mathbb{1}_{\{\tau \geq n+1\}} f(S_{n+1})\right) + E_n\left(\mathbb{1}_{\tau \leq n} f(S_\tau)\right)$$

$$\left(\{\tau \leq n\} \in \mathcal{F}_n \Rightarrow \{\tau > n\} \in \mathcal{F}_n \Rightarrow \{\tau \geq n+1\} \in \mathcal{F}_n\right)$$

$$f(S_{\tau \wedge n}) \overset{\text{Want}}{=} \mathbb{1}_{\{\tau > n\}}\left(f(uS_n)p + f(dS_n)q\right) + \mathbb{1}_{\{\tau \leq n\}} f(S_\tau)$$

$\tau \wedge n$ $\quad$ $\tau \wedge n$ $\quad$ $\tau \wedge n$

$s = S_{\tau \wedge n}$

$$\underline{f}(s) = \mathbb{1}_{\{\tau > n\}} \left(p f(us) + q f(ds) \right) + \mathbb{1}_{\{\tau \le n\}} f(s)$$

Rep th

$$f(\underline{s}) = \mathbb{1}_{s \in (l, u)} \left(p f(u\underline{s}) + q f(d\underline{s}) \right) + \mathbb{1}_{\{s \in \{l, u\}\}} f(s)$$