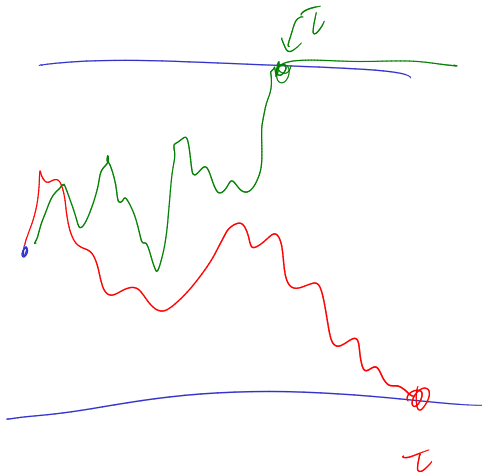


$$u^0 S_0 = \underline{u}$$

S_0

$$d^m S_0 = \underline{L}$$



Want $\underline{f}(S_{\tau \wedge n})$ to be a mg.

$$\text{Want } \underline{f}(S_{\tau \wedge n}) = E_n \left[\underline{f}(S_{\tau \wedge (n+1)}) \right]$$

try & compute.

$$E_n \left[\mathbb{1}_{\{\tau \geq n+1\}} \underline{f}(S_{n+1}) + \mathbb{1}_{\{\tau < n+1\}} \underline{f}(S_\tau) \right]$$

$$= E_n \left(\frac{1}{\{\tau \geq n+1\}} \underline{f}(S_{n+1}) \right) + E_n \left(\frac{1}{\tau \leq n} f(S_\tau) \right)$$

$$\left(\{\tau \leq n\} \in \mathcal{F}_n \Rightarrow \{\tau > n\} \in \mathcal{F}_n \Rightarrow \{\tau \geq n+1\} \in \mathcal{F}_n \right)$$

Want

$$\underline{f}(S_{\tau \wedge n}) \stackrel{\text{Want}}{=} \frac{1}{\{\tau > n\}} \left(\underline{f}(S_{\tau \wedge n}) p + \underline{f}(S_{\tau \wedge n}) q \right) + \frac{1}{\{\tau \leq n\}} \underline{f}(S_{\tau \wedge n})$$

Diagram illustrating the decomposition of the stopped process $\underline{f}(S_{\tau \wedge n})$ into a martingale part and a predictable part.

- The first term, $\underline{f}(S_{\tau \wedge n})$, is shown with a red dot under the n in the subscript, and a green arrow points to it.
- The second term, $\frac{1}{\{\tau > n\}} (\underline{f}(S_{\tau \wedge n}) p + \underline{f}(S_{\tau \wedge n}) q)$, is shown with a red dot under the n in the subscript, and a red arrow points to the $\tau \wedge n$ in the denominator.
- The third term, $\frac{1}{\{\tau \leq n\}} \underline{f}(S_{\tau \wedge n})$, is shown with a red dot under the n in the subscript, and a red arrow points to the $\tau \wedge n$ in the denominator.

$$S = \sum_{\tau \wedge \eta}$$

$$\underline{f(s)} = \frac{1}{\{\tau > u\}} \left(p f(us) + q \int f(ds) \right) + \frac{1}{\{\tau \leq u\}} f(s)$$

Rep $\hookrightarrow h$

$$\underline{f(s)} = \frac{1}{s \in (L, u)} \left(p f(\underline{us}) + q \int f(\underline{ds}) \right) + \frac{1}{\{s \in \{L, u\}\}} f(s)$$