$Y_{\tau+1} = X_\tau (1+r) \longrightarrow \boxed{Y_\tau = X_\tau}$

Cash

Stock

sell all stock. $Y_{\tau+1} = (1+r) Y_\tau = (1+r) X_\tau$

$\tau$

(random)

$Y \begin{cases} \Gamma \text{ - shares stock} \\ \text{net cash} \end{cases}$

$X \begin{cases} \Delta \text{ shares stock} \\ \text{cash.} \end{cases}$

$$\Gamma_n = \Delta_n \mathbb{1}_{\{n<\tau\}}$$

$$\mathbb{1}_{\{n=\tau\}} Y_n = \mathbb{1}_{n=\tau} \left( \Gamma_{n-1} S_n + (1+r)\big(\underbrace{Y_{n-1}}_{X_{n-1}} - \underbrace{\Gamma_{n-1}}_{\Delta_{n-1}} S_{n-1}\big) \right)$$

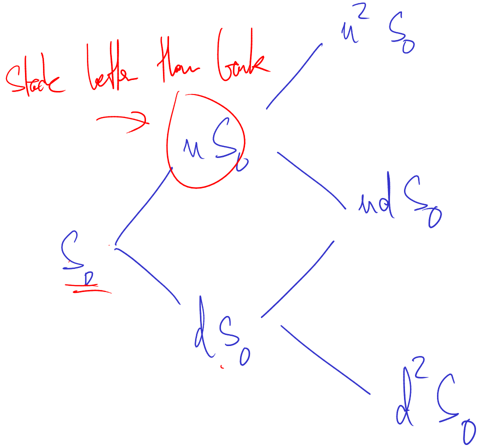
HW → Q2 b]

Goal: Start $X_0 = 0$

End at a random time $\tau$ with $X_\tau \geq 0$ & $P(X_\tau > 0) > 0$

$0 < d < \underbrace{1 + r < u}$

$X_0 = 0 \begin{cases} \to \text{stock} \\ \to \text{Bank.} \end{cases}$



Choose $\Delta_0 = 1$

$-\Delta_0 S_0$ cash in bank

Strategy: If $\omega_1 = +1$, then cash out at $n = 1$

(not a stopping time) If $\omega_1 = -1$, " " " " $n = 0$

Check: $X_\tau \geq 0$, $P(X_\tau > 0) > 0$

HW 7 Q3)

① find formula for $X_n = V_n$ = AFP before option pays.

(know how to do)

② Given $X_n$ how do we find the trading strategy?

$X_n \longrightarrow$ self fin.

$$X_{n+1} = \Delta_n S_{n+1} + (1+r)(X_n - \Delta_n S_n)$$

$\omega = (\omega', \omega_{n+1}, *)$

① $\omega_{n+1} = 1 \longrightarrow X_{n+1}(\omega', 1) = \Delta_n u S_n + (1+r)(X_n - \Delta_n S_n)$

② $\omega_{n+1} = -1 \longrightarrow X_{n+1}(\omega', -1) = \Delta_n d S_n + (1+r)(X_n - \Delta_n S_n)$

Find $\Delta_n$ : ① − ② : $X_{n+1}(\omega', 1) - X_{n+1}(\omega', -1) = \Delta_n S_n (u-d)$

$$\boxed{\Rightarrow \Delta_n = \frac{X_{n+1}(\omega', 1) - X_{n+1}(\omega', -1)}{(u-d) S_n(\omega')}}$$

$$X_{n+1} = V_{n+1} = f_{n+1}(\underline{S_{n+1}}, \underline{M_{n+1}})$$

$$X_{n+1}(\omega', 1) = f_{n+1}(u S_n(\omega'), (u S_n) \vee M_n)$$

$\Delta_n$ = some formula involving only $S_n$, $M_n$.

$$= \underline{g_n}(S_n, M_n)$$

Q4) Given $E M_\sigma = 0$ $\forall$ finite stopping time $\sigma$.

NTS $M$ is a mg

NTS $E_n M_{n+1} = M_n$ (almost surely)

$$E_n M_{n+1} \begin{cases} > M_n \\ = M_n \\ < M_n \end{cases}$$

$> M_n$ ← must happen with prob 0

$= M_n$ ← good

$< M_n$ same, happens with prob 0

Let $A = \{E_n M_{n+1} > M_n\} \in \mathcal{F}_n$

Let $\sigma = \begin{cases} n+1 & \omega \in A \\ n & \omega \notin A \end{cases}$

Q: $\sigma$ a stopping time? –

Know.

$$E M_n = E M_0 = E M_T = E\left( M_{n+1} \mathbb{1}_A + M_n \mathbb{1}_{A^c} \right)$$

$$= E E_n \left( \qquad \right)$$

$$= E\left( \mathbb{1}_A \underbrace{E_n M_{n+1}}_{> M_n \text{ on } A} + \underline{M_n} \mathbb{1}_{A^c} \right)$$

$$> E M_n \qquad (\text{if } P(A) > 0)$$