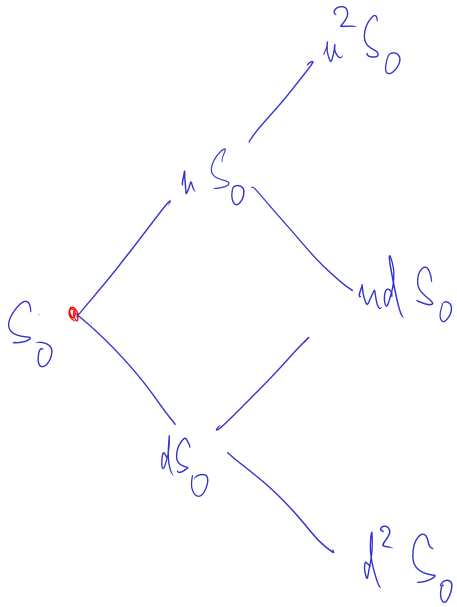
HW 7 Q 2 b:

Want a random time τ (need not be a stopping time)

& a self fin port s.t. $\underbrace{X_0 = 0, \quad X_\tau \geq 0 \quad \& \mathbb{P}(X_\tau > 0) > 0}_{\text{arbitrage.}}$

$0 < d < 1+r < u.$



$X_0 = 0$ $\begin{cases} \Delta_0 = 1 \\ \text{Cash} = -S_0 \text{ in bank.} \end{cases}$

$$\tau = \begin{cases} 1 & \omega_1 = 1 \\ 0 & \omega_1 = -1 \end{cases}$$

Q4: WTS Say $M_0 = 0$.

Assume $E M_\tau = 0$ $\forall$ finite stopping time τ.

Want M to be a mg

↳ Want $E_n M_{n+1} = M_n$.

Q: Let $\boxed{A = \{ \underbrace{E_n M_{n+1}}_{\mathcal{F}_n \text{ meas}} > \underbrace{M_n}_{\mathcal{F}_n\text{-meas}} \}}$ ← on A "stop at time $n+1$
outside A stop at time n

$\in \mathcal{F}_n$.

Let $\tau = n \mathbb{1}_{A^c} + (n+1)\mathbb{1}_A$

$\mathbb{E} M_\tau$ has to $= 0$ (by assumption)

$$\mathbb{E} M_\tau = \mathbb{E}\left(M_n \mathbb{1}_{A^c} + M_{n+1} \mathbb{1}_A \right)$$

$$= \mathbb{E}\, \mathbb{E}_n \left(M_n \mathbb{1}_{A^c} + M_{n+1} \mathbb{1}_A \right)$$

$$= \mathbb{E}\left(M_n \mathbb{1}_{A^c} + \mathbb{1}_A \mathbb{E}_n M_{n+1} \right)$$

$\boxed{\mathbb{E}_n M_{n+1} = M_n \quad \text{almost surely}}$

$$> M_n$$

$$= E\left(1_{A^c} M_n + 1_A \left(\text{something} > M_n \right) \right)$$

$$> E M_n \qquad \left(\text{if } P(A > 0) \right).$$

$$= 0$$