HW 4 Q 5 f)

Arbitrage: There is arb if $\exists$ a self fin portfolio $\ni$

(1) $X_0 = 0$

(2) $X_N \geq 0$

& (3) $P(X_N > 0) > 0$

There is no arb if for every self fin portfolio with ① $X_0 = 0$, ② $X_N \geq 0$

we have $X_N = 0$

Say X_n = wealth at time n of a self fin strat.

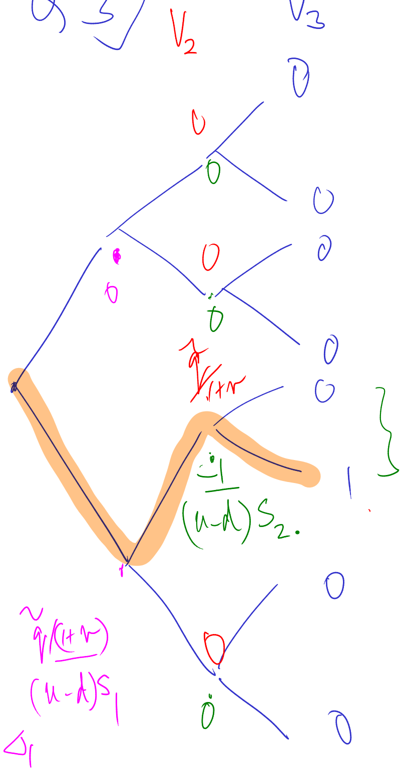
Say $X_0 = 0$ & $X_N \geq 0$

WTS: $X_N = 0$

$\Rightarrow$ $D_n X_n$ is a $\tilde{P}$ mg

$$\Rightarrow \underbrace{D_0 X_0}_{0} = \tilde{E}_0 (D_N X_N) = \tilde{E}(\underbrace{D_N}_{\geq 0} X_N)$$

HW 6 Q 3)



$\hat{\omega} = (-1, 1, -1)$

$V_3 =$ payoff of security.

$$V_2 = \frac{1}{1+r} \tilde{E}_2(V_3)$$

$$\tilde{p} = \frac{1+r-d}{u-d}$$

$$\tilde{q} = \frac{u-(1+r)}{u-d}$$

$\omega' = (\omega_1, \omega_2)$

$\uparrow$

$\omega = (\omega', \omega_3)$

$$\Delta_2(\omega) = \frac{V_3(\omega', +1) - V_3(\omega', -1)}{(u-d)\ S_2}$$

$$\Delta_1(\omega) = \begin{cases} 0 & \underline{\underline{\omega_1 = +1}} \\ \dfrac{\tilde{V}/(1+r)}{(u-d)\,S_1} & \underline{\underline{\omega_1 = -1}} \end{cases}$$

$\boxed{\text{HW6} \quad Q6}$ $\quad V_N = g\left(\sum_{n=0}^{N} S_n\right)$

Let $Y_n = \sum_{k=0}^{n} S_k$. $\quad$ Then $V_N = f_N(S_N, Y_N)$

y_n

where $f_N(s, y) = g(y)$

Hope $V_n = f_n(S_n, Y_n)$ for some fn f_n

& find a recurrence rel for f_n.

① Say $V_{n+1} = f_{n+1}(S_{n+1}, Y_{n+1})$ (true for $n+1 = N$)

want to find f_n so that $V_n = f_n(S_n, Y_n)$

Knows $V_n = \frac{1}{1+r} \tilde{E}_n V_{n+1} = \frac{1}{1+r} \tilde{E}_n \underbrace{f_{n+1}(S_{n+1}, Y_{n+1})}$

Std trick : $S_{n+1} = \underline{\underline{S_n X_{n+1}}}$, $X_{n+1} = \begin{cases} u & \omega_{n+1} = 1 \\ d & \omega_{n+1} = -1 \end{cases}$

Can we express Y_{n+1} in terms of Y_n ?

$$\hookrightarrow Y_{n+1} = \sum_{0}^{n+1} S_k = \underbrace{Y_n + S_{n+1}}_{} = Y_n + X_{n+1} S_n .$$

$$\Rightarrow V_n = \frac{1}{1+r} \tilde{E}_n f_{n+1}(S_{n+1}, Y_{n+1}) = \frac{1}{1+r} \tilde{E}_n \underline{\underline{f_{n+1}}}\left(S_n X_{n+1}, Y_n + X_{n+1} S_n\right)$$

indep lemma & write as a fn of S_n & Y_n.

$$= f_n(S_n, Y_n)$$

where $f_n(s, y)$ = some nice formula depending on s, y, f_{n+1}
(& $u, d, \tilde{p}, \tilde{q}, r$ etc)