(2a) Want to find $E_n S_{n+1}$

Let $F_{n+1}(\omega) = \begin{cases} u & \text{if } \omega_{n+1} \text{ is heads} \\ d & \text{if } \omega_{n+1} \text{ is tails.} \end{cases}$

Note $S_{n+1} = F_{n+1} S_n$

$$E_n(S_{n+1}) = E_n(F_{n+1} S_n) = S_n E_n F_{n+1}$$

$$= S_n (E F_{n+1}) \quad (\because F_{n+1} \text{ is ind of } \mathcal{F}_n)$$

$$E_n S_{n+2} = E_n \left(\underbrace{E_{n+1}} S_{n+2} \right)$$

$$= E_n \left(E F_{n+2} \right) S_{n+1}$$

$$(EF)^2 \quad S_n$$

(1b). Original Jensen from HW 1.

$$\varphi(x) \geq \varphi(a) + (x-a)\,\varphi'(a) \qquad \forall\, a,$$

$$\varphi(X) \geq \varphi(EX) + (X - EX)\,\varphi'(EX) \qquad (*)$$

Take Expectations $\Rightarrow$ Jensen

For (1b) $\varphi(X) \geq \varphi(E_n X) + (X - E_n X)\,\underline{\varphi'(E_n X)}$

Take E_n of both sides & use properties

(6) $E(X-Y)^2$ Want $\geq E(X-E_nX)^2$.

Try $E(X-E_nX+E_nX-Y)^2 = E(X-E_nX)^2 + E(E_nX-Y)^2$

$+ 2 E\left[(X-E_nX)\underbrace{(E_nX-Y)}_{\mathcal{F}_n \text{ meas}}\right]$

Note for any RV: $E\, \underline{E_n Z} = \sum_{\omega \in \Omega} E_n Z(\omega)\, f(\omega) = \sum_{\omega \in \Omega} Z(\omega) f(\omega) = \underline{\underline{E Z}}$

$$E\,(X - E_n X)(E_n X - Y) = E\, E_n (X - E_n X)\underbrace{(E_n X - Y)}$$ & simplify.

Q: $\underline{\underline{X_n}}(\omega) = \underline{\underline{\omega_{15}}}$. Is X_n adapted? NO!

For this to be adapted need X_n to be $\mathcal{F}_n$-meas $\underline{\underline{\forall n}}$

i.e. X_1 is $\mathcal{F}_1$ meas

X_2 " $\mathcal{F}_2$ " etc.

Q: