Given No arb.

$$S_0 \rightarrow \begin{array}{l} f_1 S_0 \\ f_2 S_0 \\ f_3 S_0 \end{array}$$

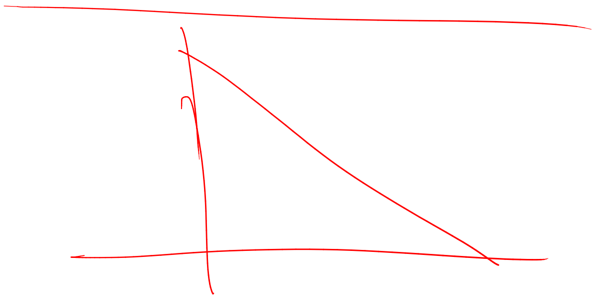
$S_0 = ?$

Want ① $\sum \tilde{\varphi}_i = 1$

② $\sum \tilde{\varphi}_i f_i = 1 + r$

~~Solve~~ $x + 2y = 1$

Find some x & y ?



Chose $y = \dfrac{100}{}$

Put $x = 1 - 2y =$

Chose $\tilde{\phi}_3 = \dfrac{t}{} \in (0, 1)$.

Find $\tilde{\phi}_1$ & $\tilde{\phi}_2$ in terms of

t, ϕ_1 ϕ_2 ϕ_3

Use Normb cond to ensure

$\tilde{\phi}_1$ & $\tilde{\phi}_2 \in (0, 1)$.

(a) OK $\boxed{f_1 < f_2 \le 1+r < f_3}$

~~$f_1 < 1+r \le f_2 < f_3$~~

$$\tilde{p}_1 = \frac{1+r - f_3 - \tilde{p}_2(f_2 - f_3)}{f_1 - f_3}$$

$$\tilde{p}_3 = 1 - \frac{\left(1+r - f_3 - \tilde{p}_2(f_2 - f_3)\right)}{f_1 - f_3} - \tilde{p}_2$$

Say $\tilde{p}_2 = \varepsilon \in (0,1)$

$$\tilde{p}_1 = \frac{f_3 - (1+r) - \varepsilon(f_3 - f_2)}{f_3 - f_1}$$ Want $\in (0,1)$.

① Can we ensure $\tilde{p}_1 > 0$? Yes :

(2) Can we ensure $\tilde{p}_1 < 1$:

$$\Leftrightarrow f_3 - (1+r) - \varepsilon(f_3 - f_2) < f_3 - f_1$$

$$\Leftrightarrow 0 < \underbrace{(1+r) - f_1}_{>0} + \varepsilon\underbrace{(f_3 - f_2)}_{>0}$$

✓

(c) $\left.\begin{array}{l}\tilde{p}_1 = \\ \tilde{p}_2 \\ \tilde{p}_3\end{array}\right\}$ one eq. $\qquad \left.\begin{array}{l}\tilde{q}_{r_1} \\ \tilde{q}_{r_2} \\ \tilde{q}_{r_3}\end{array}\right\}$ Second eq.

ONLY need

$\tilde{p}_1 \neq \tilde{q}_{r_1}$

OR $\tilde{p}_2 \neq \tilde{q}_{r_2}$

OR $\tilde{p}_3 \neq \tilde{q}_{r_3}$

$\boxed{\text{Q2f}}$

$$V_1 = (S_1 - 1)^+$$

V_0 = AFP of call opt.

$\boxed{\text{Want market to remain arb free.}}$

Start with \$0 $= X_0 = \begin{cases} \Delta_0 \text{ shares of stock} \\ \Gamma_0 \text{ call options} \\ -\Delta_0 S_0 - \Gamma_0 V_0 \rightarrow \text{cash.} \end{cases}$

$$X_1 = \underbrace{\Delta_0 S_1 + \Gamma_0 (S_1 - 1)^+ - (1+r)(\Delta_0 \, 1 + \Gamma_0 V_0)}$$

No arb: If $X_1 \geq 0$ $\forall$ die rolls. $\Rightarrow$ $X_1 = 0$ $\forall$ die rolls.

Write $X_1(1)$ = wealth at time 1 if die rolls 1.

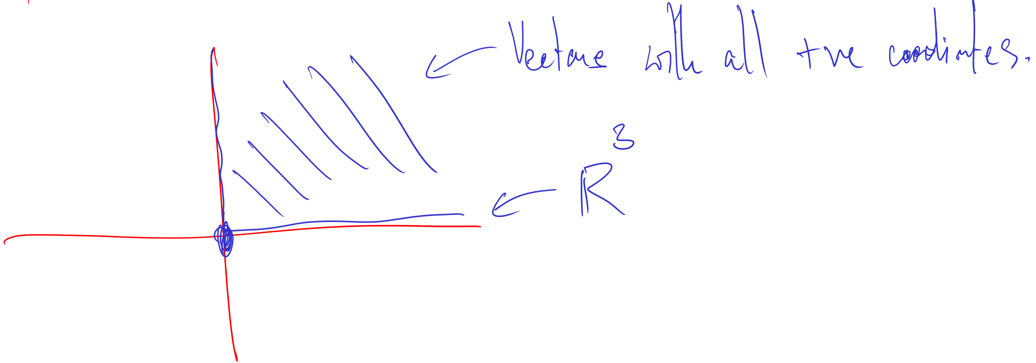
$X_1(2)$ = " " " 1 " " " 2

$X_1(3)$ = " " " " " " 3

$$X_1 = \begin{pmatrix} X_1(1) \\ X_1(2) \\ X_1(3) \end{pmatrix} \in \mathbb{R}^3 \qquad \text{(a vector).}$$

No arb: $\quad X_1(i) \geq 0 \ \forall i \in \{1, .. 3\} \Rightarrow X_1(i) = 0$

$\Leftrightarrow$



(2) 2^{nd} pic: Fix $V_0 = AFP$. Chose $\Delta_0 \in \mathbb{R}$ & $\Gamma_0 \in \mathbb{R}$ arb.

$$X_1 = \begin{pmatrix} X_1(1) \\ X_1(2) \\ X_1(3) \end{pmatrix} = \underbrace{\Delta_0 \begin{pmatrix} 1/2 \\ 1 \\ 2 \end{pmatrix} + \Gamma_0 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - (1+r)(\Delta_0 + \Gamma_0 V_0) \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}_{\text{Plane in } \mathbb{R}^3}$$

Describe $\left\{ t \begin{pmatrix} 1 \\ 2 \end{pmatrix} \,\middle|\, t \in \mathbb{R} \right\} \longrightarrow$ line.

describe $\left\{ s\begin{pmatrix}1\\2\\3\end{pmatrix} + t\begin{pmatrix}4\\5\\6\end{pmatrix} \;\middle|\; s,t \in \mathbb{R} \right\}$

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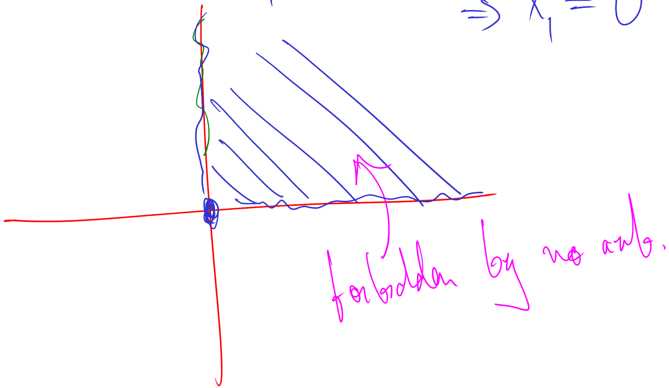
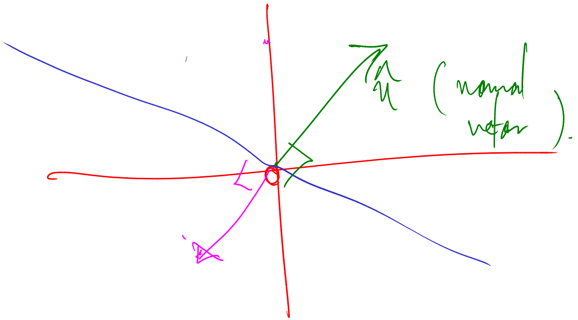
① No ~~anb~~ — X_1 has all coords ~~+ve~~ ≥ 0 $\Rightarrow X_1 = 0$



② $\{X_1 \mid s_0 \in \mathbb{R} \;\&\; t_0 \in \mathbb{R}\}$.
(plane, passing through 0.



No arb $\iff$ the plane P has a normal vector with all +ve coordinates

$$P = \left\{ \Delta_0 \begin{pmatrix} 1/2 \\ 1 \\ 2 \end{pmatrix} + \Gamma_0 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - (1+r)(\Delta_0 + \Gamma_0 V_0) \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \;\middle|\; \Delta_0 \in \mathbb{R},\ \Gamma_0 \in \mathbb{R} \right\}.$$

$$= \left\{ \Delta_0 \underbrace{\begin{pmatrix} 1/2 - (1+r) \\ 1 - (1+r) \\ 2 - (1+r) \end{pmatrix}}_{u} + \Gamma_0 \underbrace{\begin{pmatrix} -(1+r)V_0 \\ -(1+r)V_0 \\ 1 - (1+r)V_0 \end{pmatrix}}_{v.} \;\middle|\; \Delta_0, \Gamma_0 \in \mathbb{R} \right\}$$

$P = \text{span}\{u, v\}$ $\qquad \hat{n} = \pm\, \underbrace{u \times v}$

& check that $\pm \hat{n} \in$

(e) ~~X~~ Try & replicate the call. (& check whether pos or not)

$$X_0 = \Delta_0 S_0 + (X_0 - \Delta_0 S_0)$$

$$X_1 = \Delta_0 S_1 + (1+r)(X_0 - \Delta_0 S_0)$$

Q: Can you find c_0 & x_0 $\ni$ $X_1 = V_1$ $\forall$ die rolls.

2 unknowns.

3 eqns.

No soln.