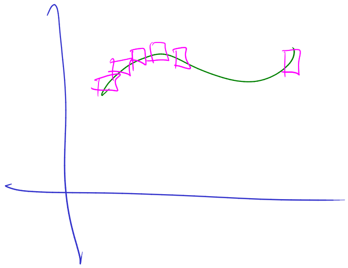
2014 Midterm Q#3

$f: [0,1] \to \cancel{\mathbb{R}}\ [0,1]$ $\mathcal{B}$ - meas

$$\Gamma = \{ (x, f(x)) \mid x \in [0,1] \}$$



Q: $\lambda^*(\Gamma) \overset{?}{=} 0$ Guess $\to 0$

Find g + g cts & $\mu\{\underline{f \neq g}\} < \varepsilon$

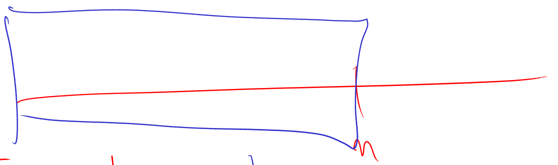
& conv by next.

~~2144~~ Q 2014 Midterm Q 4: & $\boxed{\phi_n \text{ bdd}}$

$(\phi_n) \to 0$ ptwise, ~~f cts bdd~~ $f: \mathbb{R} \to [0,1]$ $\boxed{\underline{\text{cts}}}$

Q: $\lim\limits_{n\to\infty} \int_{\mathbb{R}-\{0\}} \phi_n(t)\, \underline{f\left(\tfrac{1}{t}\right)}\, dt \overset{?}{=}$

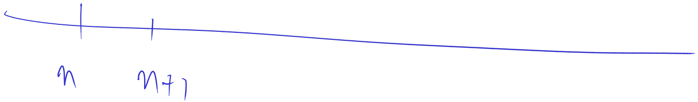
$f\left(\tfrac{1}{t}\right) \approx f(0)$



t large $\quad f\left(\tfrac{1}{t}\right) \approx f(0)$

$$\boxed{\int_0^K \phi_n(t)\, f\left(\tfrac{1}{t}\right) \xrightarrow{n\to\infty} \int_0^K 0}$$

??



$\forall\, t \geq K \qquad \left| f\left(\tfrac{1}{t}\right) - f(0) \right| < \varepsilon$

Q: $\lim\limits_{n\to\infty} \int_K^\infty \phi_n(t)\, dt = 1 - \underline{\lim\limits_{n\to\infty} \int_0^K \phi_n} = 1$

$$\to \phi_n = \begin{cases} \mathbb{1}_{[n,n+1]} & n \text{ odd} \\ n\,\mathbb{1}_{[0,1/n]} & n \text{ even} \end{cases}$$

(If ϕ_n is not odd)

$\cancel{f(t) = 1}$

$\boxed{f(t) = \cancel{n}t}$

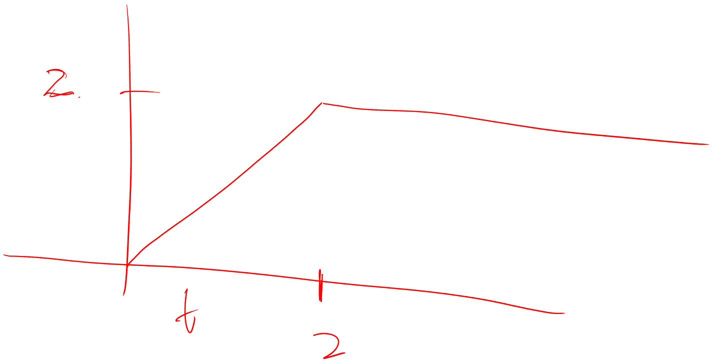
$f : \mathbb{R} \to [0,1]$

$$\lim \int_0^\infty f\left(\tfrac{1}{t}\right) \phi_n(t)\, dt \xrightarrow{n \text{ odd}} f(0)$$

$$\xrightarrow{n \text{ even}} f(0)$$

n even: $\displaystyle\int_0^{1/n} n f\left(\tfrac{1}{t}\right) dt = 2 \neq$

$f(t) =$



2013 midterm.

$\int_I f = 0 \quad \forall$ cells I. Must $f = 0$ a.e.

$U = \bigcup_1^\infty I_k$. $\int_U f = 0$?

π-sys / λ sys.

$\Lambda = \{ B \in \mathcal{B} \mid \int_B f = 0 \}$ $\quad \Lambda \supseteq \{\text{all cells}\}$.

$A_i \in \Lambda \quad A_i \subseteq A_{i+1}$, does $\bigcup_1^\infty A_i \in \Lambda$?

(Yes D.C. because $f \in \underline{L^1(\mathbb{R}^d)}$.)

Is this still true if $f \notin \underline{L^1(\mathbb{R}^d)}$.

↳ Not $\forall I$ cell, $\int_I f = 0$ (i.e f is int on I)

↳ Yes. Restrict to $[-n, n]^d$ first & send $n \to \infty$.