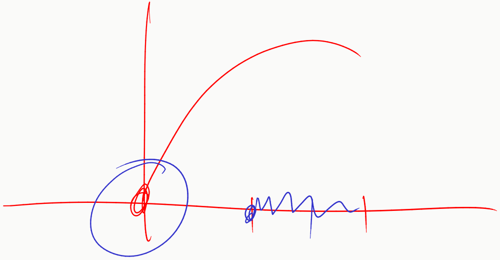
$f(x) = \sqrt{x}$



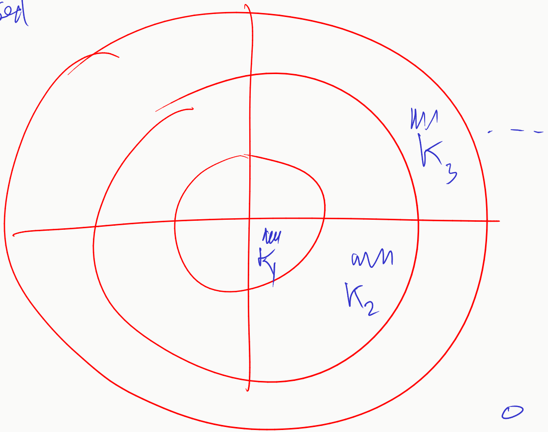
$\forall x \neq 0$

$$|f(x) - f(y)| \leq C_x |x-y| \quad \forall y \text{ close to } x$$

for $x = 0$: Can only get $|f(x) - f(y)| \leq C |x-y|^{1/2}$

Say $\overline{B}_n \subset \mathring{B}_{n+1}$ (B_n ~~open~~)

$K_n \subseteq B_{n+1} - B_n$ closed



Q: $\bigcup_{n=1}^{\infty} K_n$ closed?

$C = \bigcup_{n=1}^{\infty} K_n$

$(c_n) \to \underline{c}$, $c_n \in C \ \forall n$

NTS $\underline{c} \in C$. (Pf: $\exists n_0$ s.t. $\forall n > n_0$, $c_n \in \mathring{B}_{n_0}$

$\Rightarrow c_n \in \bigcup_{k=1}^{n_0} K_{k} \quad \forall$)

μ Regular : ① $\mu(K) < \infty \quad \forall K$ cpt

② $\forall A$, $\mu(A) = \inf\{\mu(U) \mid U \supseteq A \text{ open}\}$

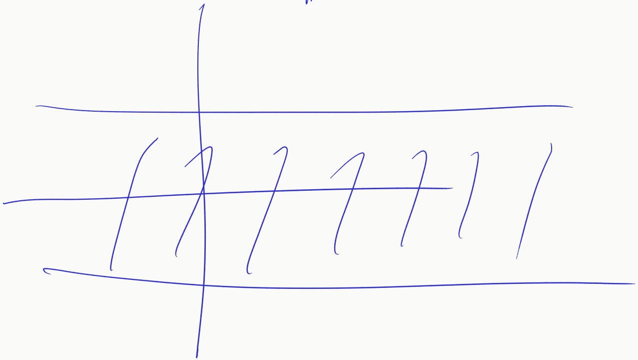
③ $\forall \underline{U}$ open, $\mu(U) = \sup\{\mu(K) \mid K \subseteq U \text{ cpt}\}$.

$A^{(\text{finite is})} \longrightarrow \exists\, C$ closed, U open $\ni \underline{\mu(U - C)} < \varepsilon$ ...

Note: If μ is finite (σ finite)

then $\forall A$, $\mu(A) = \sup\{\mu(K) \mid K \subseteq A \text{ cpt}\}$

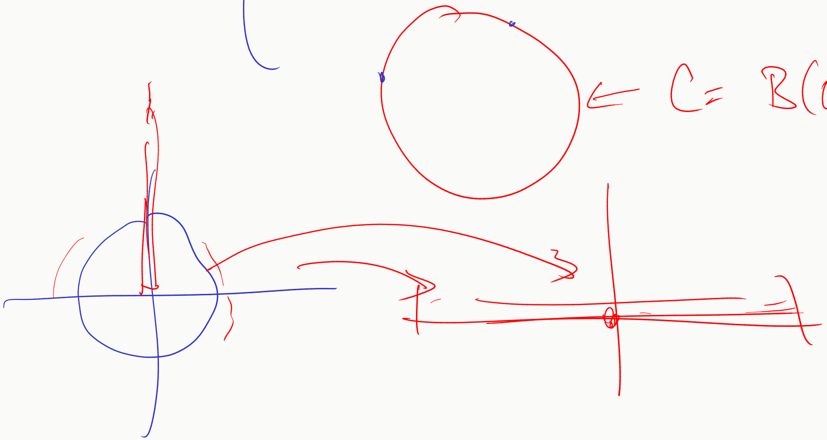
$\mathbb{R}^2$



HW 4 Q1. $\boxed{(a)}$ Convex $\Rightarrow$ $\mathcal{L}$ - meas

(b) Convex $\not\Rightarrow$ B meas

Put a non meas set here.



$C = B(0,1) \cup$ $\boxed{\text{arb subset of } \partial B(0,1)}$

C convex: Guess $\bar{C}$ is still convex (Closure $\in \mathcal{B}$)

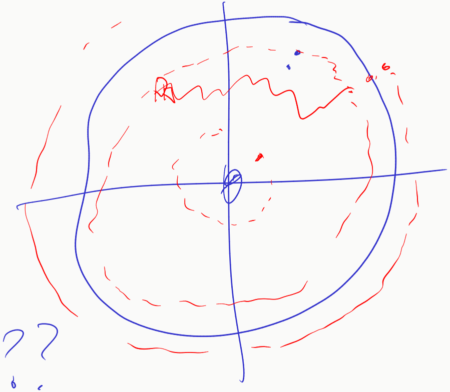
Guess $\mathring{C}$ " " " (Open $\in \mathcal{B}$)

$$\boxed{\text{Guess } \lambda(\bar{C} - \mathring{C}) = 0} \longrightarrow \text{done!}$$

(WL $0 \in C$)

Guess $(1-\varepsilon)\bar{C} \subseteq \mathring{C}$

$$(1+\varepsilon)\bar{C} \supseteq C \supseteq (1-\varepsilon)\mathring{C}$$



Q: $\lambda\left((1+\varepsilon)\bar{C} - \mathring{C}\right) \xrightarrow{\varepsilon \to 0} 0$??

(False for Half plane)

Should be true if C is [illegible].