Q! μ, ν $\quad \nu = \mu$ on $\mathcal{C}$ ← a π system.

$$\mu(C_i) = \nu(C_i) < \infty; \quad \bigcup_1^\infty C_i = X$$

Q: $\mu = \nu$ on $\sigma(\mathcal{C})$.

Q: Is $\mu = \nu$ on $\underline{C_1 \cup C_2}$?

$$\mu(C_1) + \mu(C_2) - \mu(\underbrace{C_1 \cap C_2})$$

$$= \nu(\quad) + \nu(\quad) - \nu(\quad) \quad \checkmark$$

Induct: $\mu = \nu$ on $\underbrace{\bigcup_1^N C_i}_{D_N}$ & take $\underline{\lim}$

$\boxed{Q5b}$ $\mathcal{L}(\mathbb{R}^m \times \mathbb{R}^n) \overset{?}{=} \sigma(\mathcal{L}(\mathbb{R}^m) \times \mathcal{L}(\mathbb{R}^n))$

Q: $\underline{\mathcal{L}(\mathbb{R}^2)} \overset{?}{=} \underbrace{\sigma(\mathcal{L}(\mathbb{R}) \times \mathcal{L}(\mathbb{R}))}$

Cantor $\rightarrow$ A $\rightarrow$ non meas set $\subseteq \mathbb{R}$

$$\lambda_2^*(A \times \{0\}) \leq \lambda_2^*(\mathbb{R} \times \{0\}) = 0$$

$\Rightarrow$ $A \times \{0\} \in \mathcal{L}(\mathbb{R}^2)$

NTS $\mathcal{L}(\mathbb{R}) \times \mathcal{L}(\mathbb{R}) \subseteq \mathcal{L}(\mathbb{R}^2)$

$(\Rightarrow)$ NTS $\text{If } A, B \in \mathcal{L}(\mathbb{R}) \rightarrow A \times B \in \mathcal{L}(\mathbb{R}^2)$

$$\exists \left. \begin{array}{l} F_1 \subseteq A \subseteq G_1 \\ F_2 \subseteq B \subseteq G_2 \end{array} \right\} \lambda(G_i - F_i) = 0, \quad F_i, G_i \in \mathcal{B}(\mathbb{R})$$

$$\underbrace{F_1 \times F_2}_{\mathcal{B}} \subseteq \underline{\underline{A \times B}} \subseteq G_1 \times G_2$$

Q: find $A \subseteq \mathbb{R}^2$ $\ni$ $A \notin \mathcal{L}(\mathbb{R}^2)$ | Eg $A \notin \mathcal{L}(\mathbb{R})$

$A \times \mathbb{R} \notin \mathcal{L}(\mathbb{R}^2)$

HW 2 : $S_d = \lambda$

↳ (1) $S_d(A + x) = S_d(A)$

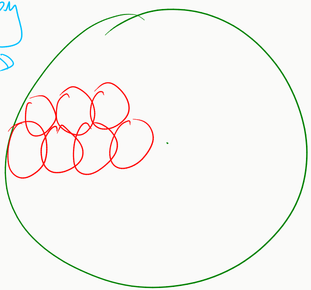
(2) $P_d(B(0,1)) = \lambda(B(0,1))$

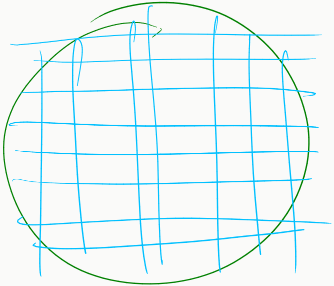
$$\boxed{\underline{\text{Claim}}: \; S_d(B(0,1)) = P_d(1)}$$

Know $S_d(B(0,1)) \geq \lambda(B(0,1))$

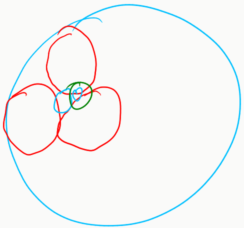
Want $S_d(B(0,1)) \leq \lambda(\quad -)$

Ch 2 Evans & Gariepy





Apollonian



$\underline{\lambda^*(A) > 0} \to A \subset \mathbb{R} \quad (A \notin \mathcal{L}(\mathbb{R}))$

$\underline{E} \subseteq A, \quad E \in \mathcal{L}(\mathbb{R}) \Rightarrow \lambda(E) = 0$

$E \subseteq A^c,\ E \in \mathcal{L}(\mathbb{R}) \Rightarrow \lambda(E) = 0$

Q3b

Q: $\kappa \in (0, \tfrac{1}{2})$

?$\exists E \subseteq \mathbb{R}$ s.t. $\dfrac{\lambda(E \cap (a,b))}{b-a} \in [\kappa, 1-\kappa]$ $\forall a<b$

Claim: $\nexists$ such sets.

Hint: Say $\exists E$

$\hookrightarrow \Lambda = \left\{ I \subseteq \mathbb{R} \;\middle|\; \dfrac{\lambda(E \cap I)}{\lambda(I)} \in [\kappa, 1-\kappa] \right\}$

$\lambda - \pi$ sys

Λ is σ-alg. $\Lambda \supseteq$ <u>all intervals</u>

$\hookrightarrow \Lambda \supseteq \mathcal{L}$

$\rightarrow E \in \Lambda$

$$\frac{\lambda(F \cap E)}{\lambda(E)} = 1$$