

$$\tilde{P}^\alpha, \tilde{P}^\beta \quad \theta \in [0, 1]$$

$\tilde{P}^\alpha, \tilde{P}^\beta$ risk neutral measures

$$\tilde{P}^\theta = \theta \tilde{P}^\alpha + (1-\theta) \tilde{P}^\beta$$

$$\theta \tilde{P}^\alpha(\Omega) + (1-\theta) \tilde{P}^\beta(\Omega) = 1$$

$A \in \mathcal{F}$

$$\theta \tilde{P}^\alpha(A) \geq 0 \quad \underline{(1-\theta) \tilde{P}^\beta(A) \geq 0}$$

$$\theta \tilde{P}^\alpha(A) \leq \theta$$

$$\tilde{E}^\alpha[D_{n+1} S_{n+1}^i] = D_n S_n^i$$

$$\tilde{E}^\theta[D_{n+1} S_{n+1}^i] \stackrel{WTS}{=} D_n S_n^i$$

$$\sum_{w_1, \dots, w_{n+1}} D_{n+1} S_{n+1}^i(w_1, w_2, \dots, w_{n+1}, w) \cdot \tilde{P}^\theta(w_1, \dots, w_{n+1}, w)$$

$$= \left[\theta \tilde{P}^\alpha(w_1, \dots, w_{n+1}, w) + (1-\theta) \tilde{P}^\beta(w_1, \dots, w_{n+1}, w) \right]$$

$$\rightarrow \theta \tilde{E}^\alpha[D_{n+1} S_{n+1}^i] + (1-\theta) \tilde{E}^\beta[D_{n+1} S_{n+1}^i]$$

4b) Arbitrage-free market

Introduce a new security \hat{S}

$$V_0 = \{ \text{arbitrage free prices for } \hat{S} \}$$

$|V_0| = 1$ iff \hat{S} is replicable

otherwise V_0 could be empty or $|V_0| \in \mathbb{N}$

$$a, b \in V_0 \quad \theta \in [0, 1]$$

Suppose

$$\theta a + (1 - \theta)b \notin V_0$$

\Rightarrow There exists
with wealth

a self-financing portfolio

$$X_1, X_0 = 0, X_N \geq 0$$

$$P(X_N > 0) > 0$$

If instead we
consider

$$\frac{a}{b} \quad X^a, X^b$$

Diagram showing a blue arrow pointing from the text "If instead we consider" to the fraction $\frac{a}{b}$ and the variables X^a, X^b .