$\tilde{\mathbb{P}}^{\alpha}, \tilde{\mathbb{P}}^{\beta}$ $\quad \theta \in [0,1]$ $\quad \tilde{\mathbb{P}}^{\alpha}, \tilde{\mathbb{P}}^{\beta}$ risk neutral measures

$$\boxed{\tilde{\mathbb{P}}^{\theta}} = \theta \tilde{\mathbb{P}}^{\alpha} + (1-\theta)\tilde{\mathbb{P}}^{\beta}$$

$$\theta \tilde{\mathbb{P}}^{\alpha}(\Omega) + (1-\theta)\tilde{\mathbb{P}}^{\beta}(\Omega) = 1$$

$A \in \mathcal{F}$

$$\theta \tilde{\mathbb{P}}^{\alpha}(A) \geq 0 \qquad \underline{(1-\theta)\tilde{\mathbb{P}}^{\beta}(A)} \geq 0$$

$$\theta \tilde{\mathbb{P}}^{\alpha}(A) \leq \theta$$

$$\tilde{\mathbb{E}}^{\alpha}[D_{n+1} S^i_{n+1}] = D_n S^i_n$$

$$\underline{\tilde{\mathbb{E}}^{\theta}[D_{n+1} S^i_{n+1}]} \overset{\text{WTS}}{=} D_n S^i_n$$

$$\theta \tilde{\mathbb{E}}^{\alpha}[D_{n+1} S^i_{n+1}] + (1-\theta)\tilde{\mathbb{E}}^{\beta}[D_{n+1} S^i_{n+1}]$$

$$= \sum_{w_1,\ldots,w_{n+1}} D_{n+1} S^i_{n+1}(w_1, w_2, \ldots, w_{n+1}, w') \cdot \underline{\tilde{\mathbb{P}}^{\theta}(w_1, \ldots, w_{n+1}, w')}$$

$$\left[\theta \underline{\tilde{\mathbb{P}}^{\alpha}(w_1, \ldots, w_{n+1}, w')} + \underline{(1-\theta)\tilde{\mathbb{P}}^{\beta}(w_1, \ldots, w_{n+1}, w')}\right]$$

4b) Arbitrage-free market

Introduce a new security $\hat{S}$

$$V_0 = \{\text{arbitrage free prices for } \hat{S}\}$$

$|V_0| = 1$ iff $\hat{S}$ is replicable

otherwise V_0 could be empty or $|V_0| \notin \mathbb{N}$

$a, b \in V_0$ $\quad \theta \in [0,1]$

Suppose $\boxed{\theta a + (1-\theta) b} \notin V_0 \Rightarrow$ There exists a self-financial portfolio with wealth X, $X_0 = 0$ $\quad X_N \geq 0$ $\quad \boxed{\mathbb{P}(X_N > 0) > 0}$

Δ

If instead we consider

a $\quad X^a$

b $\quad X^b$