Q2a
Q2b

$G = (G_0, G_1, \ldots G_N)$

$V \longrightarrow$ Snell Super mg env

$V_N = G_N$

$V_n = \max\{G_n, E_n V_{n+1}\}$

Doob Decomposition $V = \underbrace{M}_{mg} - \underbrace{A}_{pred\ inc.}$

Guess True Q2b] $V_{\tau^*} = G_{\tau^*}$ & $A_{\tau^*} = 0$? $\Longrightarrow$ τ^* is optimal ($E G_{\tau^*} \geq E G_\tau$ $\forall \tau$)

Guess ~~False~~ True 2a] τ^* optimal ? $\Longrightarrow$ $V_{\tau^*} = G_{\tau^*}$ & $A_{\tau^*} = 0$

1b: $EV_{\tau^*} = EG_{\tau^*}$

$\|$

$EM_{\tau^*} - \cancel{EA_{\tau^*}}$

OST $\|$

$EG_\tau \leq EM_\tau$

$(\because A \geq 0)$ ✓

Choose $\tau = \tau^*$

$\Rightarrow EM_\tau = EG_{\tau^*}$

2a: Know $EG_{\tau^*} \geq EG_\tau \ \forall \tau$

$\Rightarrow EV_{\tau^*} \geq EG_\tau \ \forall \tau$

$\|$

$\underbrace{EM_{\tau^*}}_{\text{OST}} - EA_{\tau^*} \geq EG_\tau \ \underbrace{\forall \tau}$

τ of my choice.

$= EM_\tau$

(for any τ of my choice)

Choose $\tau = \tau^*$

Claim: $\tau^* =$ minimal sol to the optimal stopping problem

$= \min \{ n \mid V_n = G_n \}$

Checked: $A_{\tau^*} = 0 \longleftarrow$ Needs Pf. $\Bigg\} \Rightarrow$

$V_{\tau^*} = G_{\tau^*} \longleftarrow$ Free

$M_{\tau^*} = G_{\tau^*}$

$\Rightarrow$

$E G_{\tau^*} - E A_{\tau^*}$

$> E G_{\tau^*}$

$\Rightarrow E A_{\tau^*} < 0$

$\Rightarrow A_{\tau^*} = 0$

$(\because A \geq 0)$

$\Rightarrow \quad M_{\tau^*} = V_{\tau^*}$ $\quad$ (NTS $M_{\tau^*} = V_{\tau^*} = G_{\tau^*}$)

& $\quad \mathbb{E} M_{\tau^*} = \mathbb{E} V_{\tau^*} \geq \mathbb{E} G_{\tau^*}$ $\quad$ (Know τ^* is optimal)

$\quad \overline{\parallel}$

OST $\quad \mathbb{E} M_{\sigma} \quad\quad \geq \mathbb{E} G_{\sigma} \quad \forall \sigma$ of my choice.

($\forall \sigma$ of my choice)

Choose $\sigma = \tau^*$: LHS $= \mathbb{E} M_{\tau^*} = \mathbb{E} G_{\tau^*}$

$$\Longrightarrow \quad E M_{\tau^*} = E G_{\tau^*}$$

$$\|$$

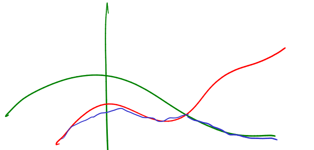
$$E V_{\tau^*} \geq E G_{\tau^*} \geq E G_{\tau^*} \quad (\because \tau^* \ \& \ \tau^* \text{ are optimal})$$

$$E G_{\tau^*} = E G_{\tau^*} = E V_{\tau^*} \geq E G_{\tau^*} \Rightarrow E V_{\tau^*} = E G_{\tau^*}$$

Also $V_{\tau^*} \geq G_{\tau^*} \Longrightarrow V_{\tau^*} = G_{\tau^*}$!

Q2e) σ^* & τ^* are both optimal

Q: is $\sigma^* \wedge \tau^*$ optimal?



Note $\boxed{2a \,\&\, b}$ $\Rightarrow$ $\boxed{\sigma^* \text{ is optimal} \iff A_{\sigma^*} = 0 \;\&\; V_{\sigma^*} = G_{\sigma^*}}$

Given σ^*, τ^* optimal $\Rightarrow$ $A_{\sigma^*} = 0$ & $V_{\sigma^*} = G_{\sigma^*}$ $\leftarrow$

& $A_{\tau^*} = 0$ & $V_{\tau^*} = G_{\tau^*}$ $\leftarrow$

$\Rightarrow A_{\sigma^* \wedge \tau^*} = 0$ & $V_{\sigma^* \wedge \tau^*} = G_{\sigma^* \wedge \tau^*}$ $\Rightarrow \sigma^* \wedge \tau^*$ is optimal!

$$\left.\begin{aligned} \sigma^* &= \min\{n \mid V_n = G_n\} \\ \tau^* &= \max\{n \mid A_n = 0\} \end{aligned}\right\} \longleftarrow \underbrace{\tau \text{ any stopping time}}$$

$$\sigma^* \leq (\tau \vee \sigma^*) \wedge \tau^* \leq \tau^*$$