

$$Q2a \left. \vphantom{\begin{matrix} Q2a \\ Q2b \end{matrix}} \right\} \rightarrow G = (G_0, G_1, \dots, G_N)$$

$$Q2b \left. \vphantom{\begin{matrix} Q2a \\ Q2b \end{matrix}} \right\} \rightarrow V \rightarrow \text{Small Supremum env} \left\{ \begin{array}{l} V_N = G_N \\ V_n = \max \{ G_n, E V_{n+1} \} \end{array} \right.$$

Doob Decomposition  $V = M - A$   
conv mg conv fund inc.

Guess True  $Q2b] V_{\tau^*} = G_{\tau^*} \& A_{\tau^*} = 0 \ ? \Rightarrow \tau^*$  is optimal  $(E G_{\tau^*} \geq E G_{\tau} \forall \tau)$

Guess ~~False~~ True  $2a] \tau^*$  optimal  $\ ? \Rightarrow V_{\tau^*} = G_{\tau^*} \& \underline{A_{\tau^*} = 0}$

$$12.6: E V_{\tau^*} = \underline{EG}_{\tau^*}$$

$\parallel$

$$EM_{\tau^*} - \cancel{EA_{\tau^*}}$$

Ost  $\parallel$

$$\underline{EG}_{\tau} \leq EM_{\tau}$$

( $\because A \geq 0$ )

Choose  $\tau = \tau^*$

$$\Rightarrow EM_{\tau} = \underline{EG}_{\tau^*}$$

$$2a: \text{Know } EG_{\tau^*} \geq EG_{\tau} \quad \forall \tau$$

$$\Rightarrow EV_{\tau^*} \geq EG_{\tau} \quad \forall \tau$$

$\parallel$

$$EM_{\tau^*} - \underline{EA_{\tau^*}} \geq EG_{\tau} \quad \forall \tau$$

$\tau$  of my choice.

$$= EM_{\tau}$$

(for any  $\tau$  of my choice)

Choose  $\tau = \tau^*$

Class :  $v^*$  = minimal sol to the optimal stopping problem  $\Rightarrow$   
 $= \min \{ m \mid \underline{V}_m = \underline{G}_m \}$   $\Rightarrow EG_{v^*} - EA_{v^*}$   
 $\geq EG_{v^*}$

Checked :  $A_{v^*} \geq 0 \leftarrow$   $\left. \begin{array}{l} \text{Nada P.f.} \\ \text{Free} \end{array} \right\} \Rightarrow$   
 $V_{v^*} = G_{v^*}$   
 $M_{v^*} = G_{v^*}$   
 $\Rightarrow EA_{v^*} \leq 0$   
 $\Rightarrow A_{v^*} = 0$   
 (∵  $A \geq 0$ )

~~✗~~

$$\Rightarrow M_{\sigma^*} = V_{\sigma^*} \quad (\text{NTS } M_{\sigma^*} = V_{\sigma^*} = G_{\sigma^*})$$

$$\& \quad E M_{\sigma^*} = E V_{\sigma^*} \geq E G_{\sigma^*} \quad (\text{K now } \sigma^* \text{ is a fund})$$

$$\text{OST} \quad \begin{array}{c} \parallel \\ E M_{\sigma} \end{array} \geq E G_{\sigma} \quad \forall \sigma \text{ of my choice.}$$

( $\forall \sigma$  of my choice)

$$\text{Choose } \sigma = \sigma^* : \text{LHS} = E M_{\sigma^*} = E G_{\sigma^*}$$

$$\begin{aligned} \rightarrow \quad EM_{\mathcal{L}^*} &= EG_{\mathcal{V}^*} \\ &= EG_{\mathcal{V}^*} \\ EV_{\mathcal{L}^*} &\geq EG_{\mathcal{L}^*} \approx EG_{\mathcal{V}^*} \quad (\because \mathcal{V}^* \text{ \& } \mathcal{L}^* \text{ are optimal}) \end{aligned}$$

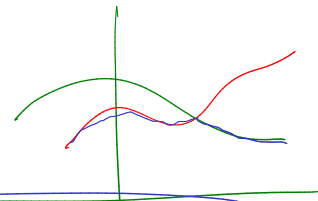
$$EG_{\mathcal{L}^*} = EG_{\mathcal{V}^*} = EV_{\mathcal{L}^*} \geq EG_{\mathcal{L}^*} \Rightarrow EV_{\mathcal{L}^*} = EG_{\mathcal{L}^*}$$

$$\text{Also } V_{\mathcal{L}^*} \geq G_{\mathcal{V}^*} \Rightarrow V_{\mathcal{L}^*} = G_{\mathcal{V}^*} !$$

Q2e

$v^*$  &  $z^*$  are both optimal

Q:  $v^* \wedge z^*$  optimal?



Note [2a & b]  $\Rightarrow$   $v^*$  is optimal  $\Leftrightarrow A_{v^*} = 0$  &  $V_{v^*} = G_{v^*}$

Given  $v^*, z^*$  optimal  $\Rightarrow A_{v^*} = 0$  &  $V_{v^*} = G_{v^*} \leftarrow$   
 &  $A_{z^*} = 0$  &  $V_{z^*} = G_{z^*} \leftarrow$

$\Rightarrow A_{v^* \wedge z^*} = 0$  &  $V_{v^* \wedge z^*} = G_{v^* \wedge z^*} \Rightarrow v^* \wedge z^*$  is optimal!

$$\left( \begin{array}{l}
 \tau^* = \min \{n \mid V_n = G_n\} \\
 \underline{\underline{c^*}} = \max \{n \mid A_n = 0\}
 \end{array} \right) \left. \begin{array}{l} \\ \\ \end{array} \right\} \leftarrow \tau \text{ any stopping time}$$

$$\tau^* \leq (\tau V_{\tau^*}) \wedge c^* \leq \underline{\underline{c^*}}$$