Last time

$G_0 = \tilde{E}\, \underbrace{D_{\tau^*} G_{\tau^*}}$

Guess $\boxed{D_n G_n}$ is a mg's $(\tilde{P})$

Q① If yes, is ~~every exercise policy optimal?~~ Yes

② How do you find G_n?

→ ①: Optimal exercise policy & τ^*

$G_N = \underline{\qquad\qquad}$

American option.

Want every exercise policy to be optimal.

G_n = intrinsic value at time n

AFP at time 0

$$\tilde{E}\left(D_\tau G_\tau\right) \leq \tilde{E}\left(D_{\tau^*} G_{\tau^*}\right) = V^0$$

$V_0^* = \tilde{E}(D_\tau G_\tau)$

for every finite stopping time τ

② If every exercise policy is optimal then

$$\Rightarrow \tilde{E}(D_\tau G_\tau) = \tilde{E}(D_{\tau^*} G_{\tau^*}) = V_0 = D_0 G_0 \quad (= G_0)$$

for every finite stopping time τ.

Note: If $D_n G_n$ is a $\tilde{P}$ mg $\Rightarrow \tilde{E}(D_\tau G_\tau) = D_0 G_0 = G_0$ (OST)

Here If $D_n G_n$ is a $\tilde{P}$ mg $\Rightarrow$ Every exercise policy is optimal!

② How do we find G_n.

Knows $D_n G_n$ is a $\tilde{P}$ mg

Knows G_N (given)

$\cancel{n = N-1}$

$$\tilde{E}_n (D_N G_N) = D_n G_n \Rightarrow \boxed{G_n = \frac{1}{D_n} \tilde{E}_n (D_N G_N)}$$

$$\boxed{M_g: E_n M_{n+1} = M_n \quad \forall n}$$

$$\left.\begin{aligned} E_n M_{n+2} &\overset{\text{tower}}{=} E_n \left(E_{n+1} M_{n+2} \right) \\ E_n M_{n+1} &= M_n \end{aligned}\right\} \Rightarrow \forall m \geq n, \ E_n M_m = M_n$$

Assume $G_n = \frac{1}{D_n}\tilde{E}_n(D_N G_N) \Rightarrow D_n G_n = \tilde{E}_n(D_N G_N)$ (*)

NTS $D_n G_n$ is a $\tilde{P}$ mg.

Pf: Want to show $D_n G_n = \tilde{E}_n(D_{n+1} G_{n+1})$

$$\tilde{E}_n(D_{n+1} G_{n+1}) = \tilde{E}_n\left(\cancel{D_{n+1}}\left(\frac{1}{\cancel{D_{n+1}}}\tilde{E}_{n+1}(D_N G_N)\right)\right)$$

$$\overset{\text{tower}}{=} \tilde{E}_n(D_N G_N) \overset{(*)}{=} D_n G_n$$

QED.

Claim: If M_n is a mg & τ is any bdd stopping time then $EM_\tau = M_0$ (OST)

Pf: Recall OST: If ① τ is a bdd stopping time & M is a mg then ② $E_n M_\tau = M_{n \wedge \tau}$

Pf ①: Apply OST with $n=0$:

$$\underset{\displaystyle EM_\tau}{\underset{\parallel}{E_0 M_\tau}} = M_{0\wedge\tau} = M_0$$

Pf ②: OST $\Rightarrow E_0 M_\tau = M_0$. Hence $E(E_0 M_\tau) = EM_0 \Rightarrow EM_\tau = M_0$

Q1 d.

$N = 3, \quad u = 2, \quad d = 1/2, \quad r = 1/4$

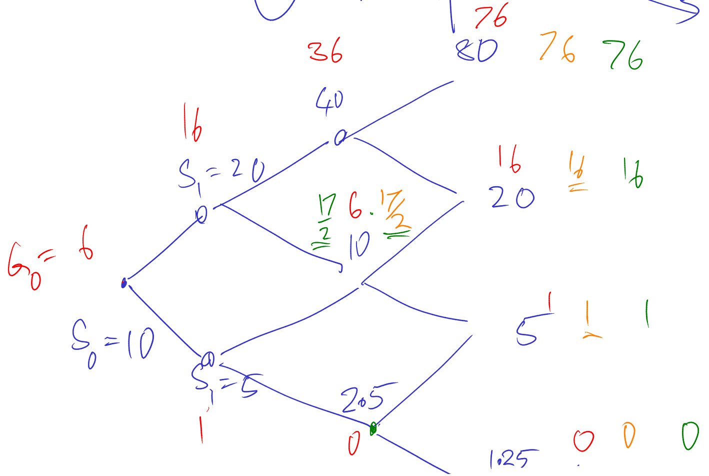
$K = 4$

AFP of ① American call $\longrightarrow$ $\underline{G_n = (S_n - K)^+}$

② European call. $\longrightarrow$ $G_N = (S_N - K)^+$

$$\left(\tilde{p} = \frac{1/2}{}, \quad \tilde{q} = \frac{1/2}{}\right)$$

$\boxed{S_0 = 10}$



Blue = S   Green : AFP of amacrine cell

Red = G   Orange : AFP of Enaptem cell