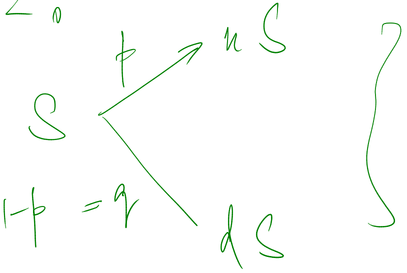
HW 8 Q2:

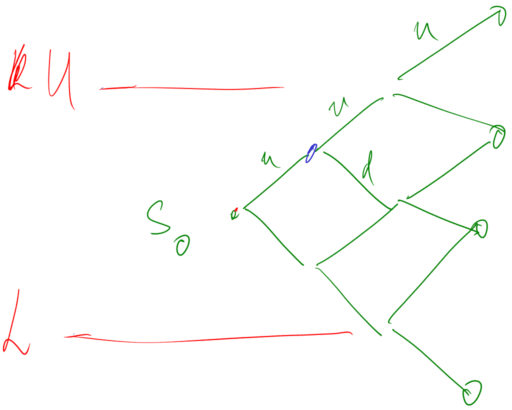


$ud = 1$

$L = d^{m_0} S_0$

$U = u^{n_0} S_0$

$\tau = \min\{n \mid S_n \notin (L, U)\}$



Q: Want $f(S_{n \wedge \tau})$ to be a mg

Goal: find f.

① Find an eq for f.

Want $E_n f(S_{(n+1)\wedge\tau}) = f(S_{n\wedge\tau})$ (*)

① Say $S_{n\wedge\tau} \in (L, U) \Rightarrow \tau > n \Rightarrow (n+1)\wedge\tau = n+1$

② $E_n f(S_{(n+1)\wedge\tau}) = E_n f(S_{n+1}) = p f(u S_n) + q f(d S_n)$

③ Want $f(\underset{s}{S_n}) = p f(u S_n) + q f(d S_n)$

$$\Rightarrow \boxed{f(s) = p f(us) + q f(ds)} \quad \forall s \in (L, U) \cap \text{range}(S_n)$$

FDE.

(4) Say $S_{n\wedge\tau} \notin (L, U) \Rightarrow \tau \leq n. \Rightarrow S_{(n+1)\wedge\tau} = S_{n\wedge\tau}$

$$\Rightarrow E_n\left(f(S_{(n+1)\wedge\tau}\,\mathbb{1}_{\{\tau\leq n\}}\right) = E_n\, f(S_{n\wedge\tau})\,\mathbb{1}_{\{\tau\leq n\}}$$

$$= f(S_{n\wedge\tau})\,\mathbb{1}_{\{\tau\leq n\}}$$

Q: Given the sol of 2b & 2c, How do you do $\boxed{2d}$??

H/Q: find f so that $f(L)=0$, $f(U)=1$ & $f(S_{\tau\wedge n})$ is a mg

Given this fn f, Find $P(S_\tau = U)$

$$E\, \underbrace{f(S_\tau)}_{S_\tau = U \text{ or } L.} \overset{OST}{=} f(S_0)$$

$$\cancelto{1}{f(U)}\, P(S_\tau = U) + \cancelto{0}{f(L)}\, P(S_\tau = L)$$

$$= P(S_\tau = U)$$

Q3a) American option: Intrinsic value $G = (G_0, G_1, \cdots G_N)$.

Let $V_N = G_N$

$$V_n = \max \left\{ G_n, \frac{1}{1+r} \tilde{E}_n V_{n+1} \right\}$$ ← going to be the AFP

$$\tau^* = \min \{ n \mid V_n = G_n \}.$$ ← going to be the minimal optimal exercise time.

$X_n = V_n + A_n$ & $A_n = \sum_{k=0}^{n-1} \left(G_k - \frac{1}{1+r} \tilde{E}_k V_{k+1} \right)^{+} (1+r)^{n-k}$

3a $\longrightarrow$ $X_n = \frac{1}{1+r} \tilde{E}_n X_{n+1}$ ($\Rightarrow D_n X_n$ is a mg under $\tilde{P}$

$\Rightarrow X =$ wealth of a self fin Portfolio)

b, c Show: $\underbrace{X_n \geq G_n}_{} \; \& \; X_{\tau^*} = G_{\tau^*}$

$\longrightarrow$