

$\Delta_n$  amount of shares to buy at time  $n$  to replicate  $X$

$$\Gamma_n = \Delta_n 1_{\{\tau > n\}}$$

$$\Gamma_0 = \begin{cases} \Delta_0 & \text{if } \tau > 0 \leftarrow \\ 0 & \text{if } \tau = 0 \leftarrow \end{cases}$$

$$X_1 = \frac{S_0 \Delta_0 + (X_0 - \Delta_0) \cdot (1+r)}$$

$$\{\tau > 0\} = \{\tau \geq 1\}$$

time  $m$   $Y_m = X_0 1_{\{\tau=0\}} (1+r)^m + X_1 1_{\{\tau=1\}} (1+r)^{m-1}$

$\tau \leq N$  a.s.  $+ \dots + X_{m-1} 1_{\{\tau=m-1\}} (1+r)^1 + X_m 1_{\{\tau \geq m\}} (1+r)^0$

$$\rightarrow Y_1 = S_0 \Gamma_0 + (X_0 - \Gamma_0) (1+r)$$

$$= 1_{\{\tau=0\}} [S_0 \Gamma_0 + (X_0 - \Gamma_0) (1+r)] + 1_{\{\tau \geq 1\}} [S_0 \Delta_0 + (X_0 - \Delta_0) (1+r)]$$

$$Y_N = \sum_{n=0}^N \underbrace{1_{\{\tau=n\}} X_n}_{=} (1+r)^{N-n}$$

$$= X_\tau \cdot (1+r)^{N-\tau}$$

$$Z = \sum_{i \in \underline{Z(\Omega)}} i 1_{\{Z=i\}}$$