HW 4 Q1

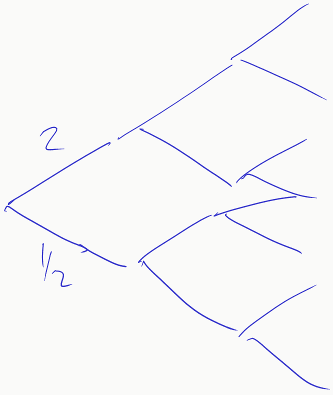
(b)

Binom model

$u = 2$

$d = \frac{1}{2}$

$r = \frac{1}{4}$



RNM $\longrightarrow$ compute $\tilde{p}_1 = 1/2$

$\tilde{q}_1 = 1/2$

$\tilde{E} S_1$ $\quad$ $\tilde{E} S_2$ $\quad$ $\tilde{E} S_3$

$$\tilde{E} S_1 = \frac{5}{4} S_0$$

$$\tilde{E} S_2 = \frac{25}{16} S_0$$

$$\tilde{E} S_3 = \frac{125}{64} S_0$$

Option 1 $\rightarrow$ find all values taken on by S_0
& the probs & sum (good for practice)

Option 2: $\left(1+\frac{1}{4}\right)$

Find a formula for $\tilde{E} S_n \stackrel{\text{Claim}}{=} (1+\underset{\substack{\downarrow \\ 1/4}}{r})^n S_0$

Pf:

Know

$\rightarrow D_n S_n$ is is a mg UNDER $\tilde{P}$

$D_n = (1+r)^{-n}$ (discount factor)

$\Rightarrow \forall n \quad \tilde{E}(D_n S_n) = \tilde{E} D_0 S_0 = S_0$

$\Rightarrow (1+r)^{-n} \tilde{E} S_n = S_0$ $\boxed{\Rightarrow \tilde{E} S_n = (1+r)^n}$

HW4

$\boxed{Q\,2\,c}$ Are coin tosses ind under $\tilde{P}$

↳ X & Y ind under P

? ⇒ X & Y " " $\tilde{P}$ (NO)

Q: $p \longrightarrow$ PMF of N $\boxed{\text{ind}}$ coin tosses.

↳ $p(\omega) = p(\omega_1, \omega_2, \cdots \omega_N) = p_1(\omega_1)\, p_2(\omega_2) \cdots p_N(\omega_N)$

Q: X, Y 2 RVs, X, Y $\boxed{\text{ind.}}$

$$P(X=x, Y=y) = P(X=x)\,P(Y=y) \quad \forall x, y$$

$N=2$: $p(\omega_1, \omega_2)$:

$(\omega = (\omega_1, \omega_2))$

$X(\omega) = \omega_1$ (RV $\rightarrow$ corresponds to 1^{st} coin toss)

$Y(\omega) = \omega_2$ (" " " 2^{nd} " ")

Pick $\hat{\omega} \in \Omega$. $\hat{\omega} = (\hat{\omega}_1, \hat{\omega}_2)$

$$P(X=\hat{\omega}_1 \;\&\; Y=\hat{\omega}_2) = P\{\omega\in\Omega \mid X(\omega)=\hat{\omega}_1,\ Y(\omega)=\hat{\omega}_2\}$$

$$= P\{\omega\in\Omega \mid \omega_1=\hat{\omega}_1,\ \omega_2=\hat{\omega}_2\}$$

$$= P\{\hat{\omega}\} = \underline{\underline{p(\hat{\omega}_1,\hat{\omega}_2)}}$$

Assumed X & Y are ind

$$\Downarrow$$

$$= \underbrace{P(X=\hat{\omega}_1)}_{p_1(\hat{\omega}_1)}\, P(Y=\hat{\omega}_2)$$

$$p_1(\hat{\omega}_1)\cdot p_2(\hat{\omega}_2)$$