

Claim: X a R.V on $\Omega \rightarrow \omega$ dice rolls

$n \leq N$. $\mathcal{F}_n \rightarrow$ info from first n die rolls.

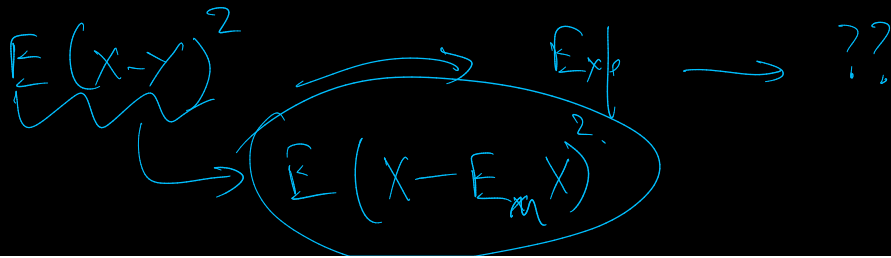
$E_n X \rightarrow$ cond exp of X given \mathcal{F}_n

$Y \rightarrow \mathcal{F}_n$ meas.

Claim: $E(X - E_n X)^2 \leq E(X - Y)^2$

(Intuition \rightarrow think of $(E(Y-Z)^2)^{1/2}$ as a measure
of the "distance" between 2 RV's Y & Z

"Amongst all E_n meas RV's Y ,
 $E_n X$ is the "best approx" to X ".



$$E(X-Y)^2 = E\left(\underbrace{(X - E_n X)}_{\text{red}} + \underbrace{(E_n X - Y)}_{\text{red}}\right)^2$$

have shown this!
↓ is 0 below!

$$= E(X - E_n X)^2 + E(E_n X - Y)^2 + \boxed{2E(X - E_n X)(E_n X - Y)}$$

What we want

≥ 0

Going to use (2) properties of E_n to evaluate

↑
???

(class today)

① Y & Z 2 RV's.

Y is E_n meas.

$Z \rightarrow$ any thing \leftarrow

Then $E_n(Y|Z) = Y|E_n Z$

$E_n Y = \cancel{EY}$

① |

"best approx of Y by an F_n meas RV"

$= Y$

② ||||

$\cancel{E_n(Y|Z)} = Y|E_n Z$

RV

(FALSE!)

\mathbb{E}
② $Z \rightarrow$ any RV. Q: $E_n Z \rightarrow \mathcal{F}_n$ meas RV

What is $E(E_n Z)$? Claim: $E \underline{\underline{E_n Z}} = E Z$

① $E_n Z$ is an \mathcal{F}_n meas RV.

② For every $A \in \mathcal{F}_n$, $\sum_{\omega \in A} E_n X(\omega) p(\omega) = \sum_{\omega \in A} X(\omega) p(\omega)$

Put A = \Omega ($\because \Omega \in \mathcal{F}_n \forall n$ incl $n=0$)

$$\Rightarrow E(\underline{E_m X}) = \underline{E X}$$

Term in Box:

$$2 E(X - E_m X)(E_m X - Y)$$

$$E[(X - E_m X)(E_m X - Y)] = E \left[\underbrace{E_m}_{\text{Term in Box}} (X - E_m X)(E_m X - Y) \right]$$

$$E_m \text{-meas} = E \left(\underbrace{(E_m X - Y)}_{\text{Term in Box}} \underbrace{(E_m(X - E_m X))}_{\text{Term in Box}} \right)$$

L

$$\left(E_n X - \underbrace{E E_n X}_{E_n X} \right)$$

0