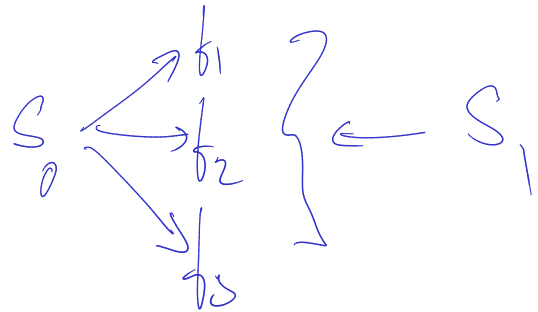


Q2a: $0 < f_1 < f_2 < f_3$



$r > -1$

Q: When is this
auf free

Guess $f_1 < 1+r < f_2$

Reason: \rightarrow Write fur

X_0 \rightarrow Cash
 \rightarrow Stake (Δ_0 shares).

$$X_1 = (1+r)(X_0 - \Delta_0 S_0) + \Delta_0 S_1$$

$$= (1+r)X_0 + \Delta_0(S_1 - (1+r)S_0)$$

Start with 0

Assume $X_1 \geq 0 \Leftrightarrow \Delta_0(S_1 - (1+r)S_0) \geq 0$

No arbitrage \Leftrightarrow If $X_0 = 0$ & $X_1 \geq 0 \Rightarrow X_1 = 0$

- ① If $S_1 - (1+r)S_0$ is always $\geq 0 \rightarrow$ have arbitrage (if choose $\Delta_0 > 0$)
- ② " " " " $\leq 0 \rightarrow$ " (if $\Delta_0 < 0$)

③ No arbitrage $\Leftrightarrow S_1 - (1+r)S_0$ is not const sign.

$$\Leftrightarrow f_1 < 1+r < f_3$$

Q26 Markt has no arbitrage. find $\tilde{p}_i \in (0,1)$ +

① $\sum \tilde{p}_i = 1$ & ② $\sum \tilde{p}_i \underline{f}_i = (1+r)$

③ $\tilde{p}_2 = c$

$$\tilde{p}_1 f_1 + \underline{f}_2 + (1 - \tilde{p}_1 - c) \underline{f}_3 = 1+r$$

$$\Leftrightarrow \tilde{p}_1 (f_1 - \underline{f}_3) = 1+r - (1-c)\underline{f}_3 - c\underline{f}_2$$

$$\Leftrightarrow \tilde{r}_1 = \frac{(1-\varepsilon)\bar{b}_3 + \varepsilon b_2 - (1+r)}{b_3 - b_1}$$

Want
 $\varepsilon \in (0, 1)$

Can choose ε . Try an angle $\tilde{r}_1 > 0$

Know $0 < b_1 < b_2 < b_3$
 and $b_1 < 1+r < b_3$

$$b_3 - (1+r) > 0$$

Want $b_3 - (1+r) > \varepsilon(b_3 - b_2) \Leftrightarrow \varepsilon < \frac{b_3 - (1+r)}{b_3 - b_2}$

Choose any $\varepsilon \in (\quad , \quad)$

$$\frac{b_3 - (1+r)}{b_3 - b_2}$$

& write $\vec{v}_1, \dots, \vec{v}_3$ in terms of α_n

$$Q2f \rightarrow f_1 = \frac{1}{2}, f_2 = 1, f_3 = 2, \quad n > 0$$

$$V_1 = (\zeta_1 - 1)^+ \quad \zeta_0 = 1$$

$$V_1 = \begin{cases} 1 & \text{if } \text{roll } 3 \\ 0 & \text{if } \text{roll } 1, 2. \end{cases}$$

Find V_0 . \rightarrow Q2c \rightarrow Cant replicate.

Remark #1) If you can replicate \rightarrow AFP is unique & = initial capital.

② || " CANT " \rightarrow " " NOT "

\rightarrow find all poss values for V_0 . \rightarrow find $\tilde{p}_1, \tilde{p}_2, \tilde{p}_3 \rightarrow$

Knows:

$$ES_1 = (1+r) \frac{C}{r} \text{ \& } \sum \tilde{p}_i = 1$$

Linear Extran market is arbitrage free!

Start with $X_0 = 0$ $\left\{ \begin{array}{l} \Delta_0 \text{ shares of Stock} \\ \Gamma_0 \text{ options (valued at } \underline{V_0}) \\ -\Delta_0 S_0 - \Gamma_0 V_0 \text{ Cash} \end{array} \right.$

Then $X_1 = \text{wealth at time 1} = (1+r) (-\Delta_0 S_0 - \Gamma_0 V_0) + \Delta_0 S_1 + \Gamma_0 V_1$

need market to remain arbitrage free
i.e. If $X_1 \geq 0$
(forall)

then $X_1 = 0$

$$X_1 = \Delta_0 (S_1 - (1+r)S_0) + \Gamma_0 (V_1 - (1+r)V_0)$$

roll 1: $X_1 = \Delta_0 \left(\frac{1}{2} - (1+r) \right) - \Gamma_0 (1+r) V_0 \geq 0$

roll 2: $X_1 = \Delta_0 (1 - (1+r)) - \Gamma_0 (1+r) V_0 \geq 0$

roll 3: $X_1 = \Delta_0 (2 - (1+r)) + \Gamma_0 (1 - (1+r)V_0) \geq 0$

Can choose Δ_0 & Γ_0 arbitrarily.