1. Syllabus Overview

- Class website and full syllabus: http://www.math.cmu.edu/~gautam/sj/teaching/2020-21/370-dtime-finance
- TA's: Lily Chen <huipingc@andrew.cmu.edu>, Jose Olvera <joseluim@andrew.cmu.edu>.
- Homework Due: Every Wednesday, before class (on Gradescope)
- Midterms: Wed Sep 30, 5th week, and Wed Nov 4th, 10th week (self proctored, can be taken any time)
- **Zoom lectures:**
  - ▷ Please enable video. (It helps me pace lectures).
  - ▷ Mute your mic when you're not speaking. Use headphones if possible. Consent to be recorded.
  - ▷ If I get disconnected, check your email for instructions.
- **Homework:**
  - ▷ Good quality scans please! Use a scanning app, and not simply take photos. (I use Adobe Scan.)
  - ▷ 20% penalty if turned in within an hour of the deadline. 100% penalty after that.
  - ▷ Bottom 20% homework is dropped from your grade (personal emergencies, other deadlines, etc.).
  - ▷ Collaboration is encouraged. Homework is not a test – ensure you learn from doing the homework.
  - ▷ You must write solutions independently, and can only turn in solutions you fully understand.
- **Exams:**
  - ▷ Can be taken at any time on the exam day. Open book. Use of internet allowed.
  - ▷ Collaboration is forbidden. You may not seek or receive assistance from other people. (Can search forums; but may not post.)
  - ▷ Self proctored: Zoom call (invite me). Record yourself, and your screen to the cloud.
  - ▷ Share the recording link; also download a copy and upload it to the designated location immediately after turning in your exam.

- **Academic Integrity**
  - ▷ Zero tolerance for violations (automatic **R**).
  - ▷ Violations include:
    - Not writing up solutions independently and/or plagiarizing solutions
    - Turning in solutions you do not understand.
    - Seeking, receiving or providing assistance during an exam.
    - Discussing the exam on the exam day (24h). Even if you have finished the exam, others may be taking it.
  - ▷ All violations will be reported to the university, and they may impose additional penalties.
- **Grading:** 30% homework, 20% each midterm, 30% final.

2. Replication, and Arbitrage Free Pricing

- Start with a *financial market* consisting of traded assets (stocks, bonds, money market, options, etc.)
- We model the price of these assets through random variables (stochastic processes).
- **No Arbitrage Assumption:**
  - ▷ In order to make money, you have to take risk. (Can't make something out of nothing.)
  - ▷ There doesn't exist a trading strategy with $X_0 = 0$, $X_n \geq 0$ and $\boldsymbol{P}(X_n > 0) > 0$.
- Now consider a non-traded asset Y (e.g. an option). How do you price it?
- *Arbitrage free price:* V_0 is the arbitrage free price of Y, if given the opportunity to trade Y at price V_0, the market remains arbitrage free.

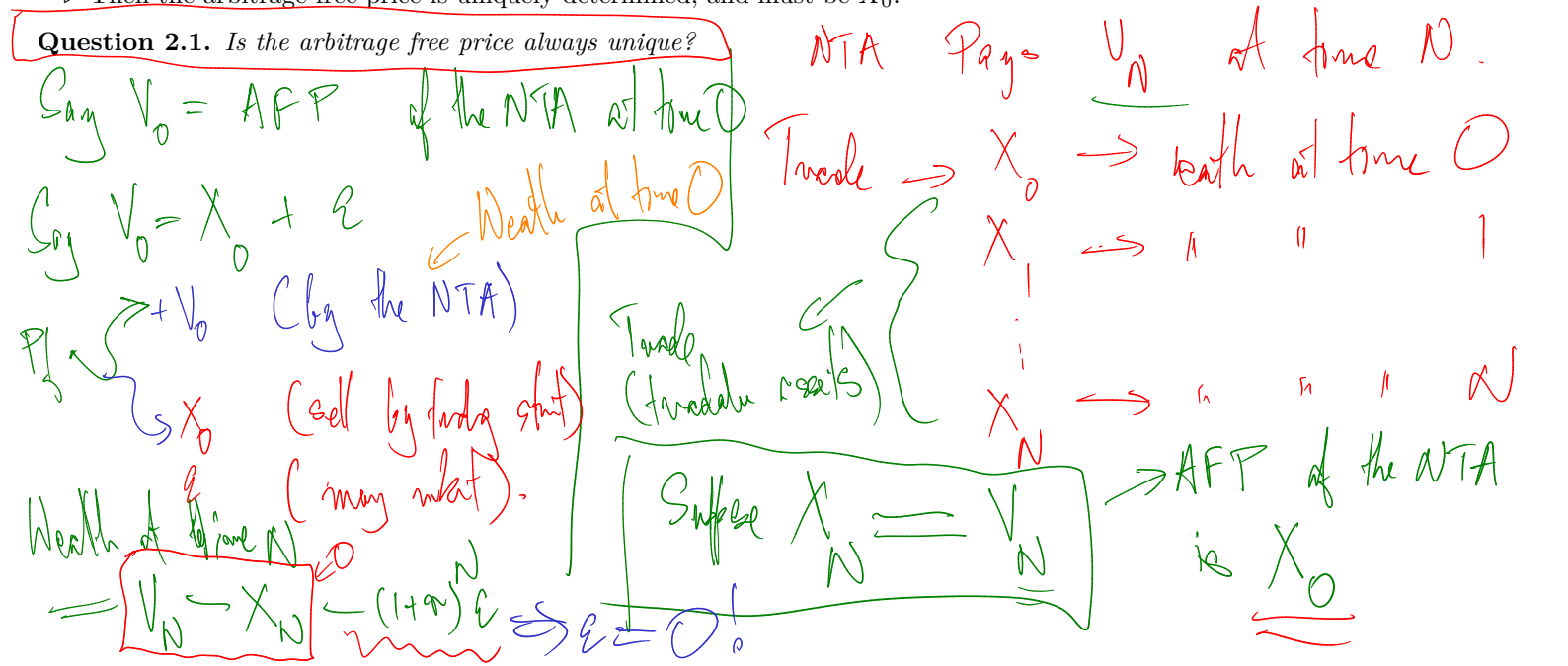
Market. ← { ① Many Market. ② Stock.

NTA: Call option → at mat time $(S_N - K)^+$

stock price, strike, mat time.

- How do you compute the arbitrage free price? Replication:
  - ▷ Say the non-traded asset pays V_N at time N (e.g. call options).
  - ▷ Say you can *replicate* the payoff through a trading strategy $X_0, \ldots, X_N = V_N$ (using only traded assets).
  - ▷ Then the arbitrage free price is uniquely determined, and must be X_0.

Question 2.1. *Is the arbitrage free price always unique?*



Theorem 2.2. *The arbitrage free price is* unique *if and only if there is a replicating strategy! In this case, the arbitrage free price is exactly the initial capital of the replicating strategy.*

Proof. We already proved that if a replicating strategy exists then the arbitrage free price is unique. The other direction is harder, and will be done later. □

Question 2.3. *If a replicating strategy exists, must it be unique?*

NO.

But initial wealth of the replicating strat is unique (AFP!!)