Stochastic Calculus for Finance I: Midterm.

2020-02-06, Pittsburgh

- *This is a closed book test. No electronic devices may be used. You may not give or receive assistance.*
- *You have 90 minutes. The exam has a total of 5 questions and 25 points.*
- *The questions are roughly ordered by difficulty. Good luck.*

In this exam W always denotes a standard Brownian motion, and the filtration $\{\mathcal{F}_t \,|\, t \geqslant 0\}$ (if not otherwise specified) is the Brownian filtration.

[5] 1. Compute $\boldsymbol{E}\Big(\int_0^t s^2 W(s)\, dW(s)\Big)^2$, and express it terms of t, without involving W expectations or integrals.

[5] 2. Let $X(t) = te^{2W(t)} + \displaystyle\int_0^t \frac{1}{1+W(s)^2}\, ds$. Find $[X,X](t)$. Express your answer in the form $[X,X](t) = \int_0^t f(s,W(s))\, ds + \int_0^t g(s,W(s))\, dW(s)$ for two functions f and g you explicitly find the formula for.

[5] 3. Let $0 \leqslant s < t$. Compute $\boldsymbol{E}[W(s)^2 W(t)^2]$. Express your answer in terms of s and t without involving W, expectations or probabilities.

[5] 4. Let $0 \leqslant s < t$. Compute $\boldsymbol{E}\big[\exp\big(2W(s)W(t)\big) \,\big|\, \mathcal{F}_s\big]$. Your answer may involve s, $W(s)$ and t, but not any expectations or integrals.

[5] 5. Find $\boldsymbol{E}\Big[W(t)\displaystyle\int_0^t s^2\, dW(s)\Big]$. Express your answer in terms of t, without involving W, expectations or probabilities.

HINT: First try and express $\int_0^t s^2\, dW(s)$ in the form $f(t,W(t)) + \int_0^t g(s,W(s))\, ds$ for two (non-random) functions f and g.