Stochastic Calculus for Finance I: Midterm.

2019-02-07

- *This is a closed book test. No electronic devices may be used. You may not give or receive assistance.*
- *You have 90 minutes. The exam has a total of 5 questions and 25 points.*
- *The questions are roughly ordered by difficulty. Good luck and .*

In this exam W always denotes a standard Brownian motion, and the filtration $\{\mathcal{F}_t \,|\, t \geqslant 0\}$ (if not otherwise specified) is the Brownian filtration.

5 1. Define the process X by $X(t) = \int_0^t W(s)^2\, dW(s)$. Compute $\boldsymbol{E}X(t)$ and $\boldsymbol{E}[X(t)^2]$. Express your answers as explicit functions of t without involving W, expectations or integrals.

5 2. Let $X(t) = te^{-2W(t)}$. Find a martingale, M, and an adapted process with finite first variation, B, such that $X(t) = M(t) + B(t)$. (You may leave your answers as a combination of Riemann and or Itô integrals.)

5 3. Let $X(t) = \exp(-tW(t)^2) + \int_0^t W(s)^2\, ds - \int_0^t \exp(W(s))\, dW(s)$. Compute $[X, X](t)$. (You may leave your answer as an integral.)

5 4. Let X and Y be two independent standard normal random variables. Let $Z = X + Y$, and find $\boldsymbol{E}(X \mid Z)$. Your final answer may involve X and Y but should not involve any expectations or integrals.

5 5. Let $M(t) = \int_0^t rW(r)\, dW(r)$. For $0 \leqslant s < t$, compute $\boldsymbol{E}(M(t)^2 \mid \mathcal{F}_s)$.