21-268 Multidimensional Calculus: Midterm 1.

2020-02-12

- *This is a closed book test. No electronic devices may be used. You may not give or receive assistance.*
- *You have 50 minutes. The exam has a total of 5 questions and 25 points.*
- *You may use any result proved in class or any regular homework problem* ***PROVIDED*** *it is* independent *of the problem you want to use the result in. (You must also* ***CLEARLY*** *state the result you are using.)*
- *The questions are roughly in increasing order of difficulty. Good luck* $\ddot\smile$.

5 1. Let f be a differentiable function of two variables, y_1, y_2. Suppose the variables y_1, y_2 are each in turn differentiable functions of three variables x_1, x_2, x_3. Finally suppose each variable x_1, x_2, x_3 is a differentiable function of one variable t. Find $\frac{df}{dt}$ in terms of the partial derivatives of f with respect to y_1, y_2, the partial derivatives of y_1, y_2 with respect to x_1, x_2, x_3 and the derivatives of x_1, x_2, x_3 with respect to t. [Since there are many variables here, to avoid ambiguity please denote derivatives as $\partial f/\partial y_i$ etc. and not simply $\partial_i f$.]

5 2. Suppose $f\colon \mathbb{R}^d \to \mathbb{R}$ is differentiable at a point $a \in \mathbb{R}^d$. Let $v \in \mathbb{R}^d$ be a non-zero vector. Does the directional derivative of f in the direction v exist at the point a? Prove it. [This is something we did in class. Please provide a direct, complete proof here without simply quoting the result from class.]

5 3. Let $f(x_1, x_2) = 2x_1 - 3x_2^2$. Show directly using the ε-δ definition that $\lim\limits_{x\to(1,2)} f(x) = -10$.

5 4. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x) = x_1^{2/3}x_2^{2/3}$. Is f differentiable at the point $(0, 1)$? Prove your answer.

5 5. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x) = x_1^{2/3}x_2^{2/3}$. Is f differentiable at the point $(0, 0)$? Prove your answer.