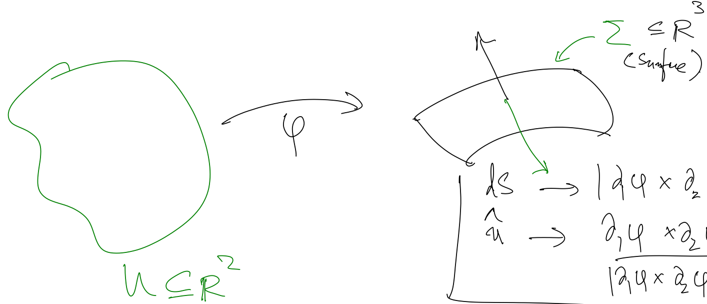


$$dS \longrightarrow |\partial_1\varphi \times \partial_2\varphi| \, dA$$

$$\hat{n} \longrightarrow \frac{\partial_1\varphi \times \partial_2\varphi}{|\partial_1\varphi \times \partial_2\varphi|}$$

① $\rho : \Sigma \to \mathbb{R}$ then $\displaystyle\int_\Sigma \rho \, dS = \int_U \rho\circ\varphi \, |\partial_1\varphi \times \partial_2\varphi| \, dA$

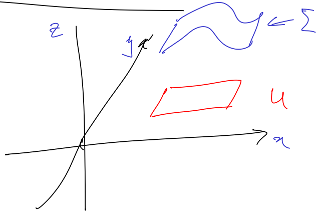
② $F : \Sigma \to \mathbb{R}^3$ & Σ is oriented ($\hat{n}$ is given) then $\dfrac{\partial_1\varphi \times \partial_2\varphi}{|\partial_1\varphi \times \partial_2\varphi|}$ is a unit normal vect $\Rightarrow \hat{n} = \pm \dfrac{\partial_1\varphi\times\partial_2\varphi}{|\partial_1\varphi\times\partial_2\varphi|}$

$$\Rightarrow \int_\Sigma F\cdot\hat{n}\, dS = \int_U F\circ\varphi \cdot (\partial_1\varphi \times \partial_2\varphi)\, dA \quad \left(\text{if } \hat{n} = +\frac{\partial_1\varphi\times\partial_2\varphi}{|\partial_1\varphi\times\partial_2\varphi|}\right)$$

Q1: $U \subseteq \mathbb{R}^2$ $\quad f: U \to \mathbb{R}$ $\quad C^1$

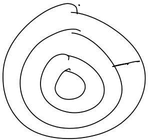
$$\text{Area}(\Sigma) = \int_U \sqrt{1 + |\nabla f|^2}\, dA$$

$$\hookrightarrow \text{Area}(\Sigma) = \int_\Sigma 1\, dS$$



Param of Σ: Need $\varphi : U \longrightarrow \Sigma$

$x, y \in U$, $\quad \varphi(x,y) = \begin{pmatrix} x \\ y \\ f(x,y) \end{pmatrix} \in \Sigma$



$$\partial_1 \varphi = \begin{pmatrix} 1 \\ 0 \\ \partial_x f \end{pmatrix} \qquad \partial_2 \varphi = \begin{pmatrix} 0 \\ 1 \\ \partial_y f \end{pmatrix}$$

adjugate
adjunct
transpose of cof

$A = (a_{ij})$

Q: Formula for A^{-1}: $\frac{1}{\det(A)} \boxed{\operatorname{cof}(A)^T}$ adj(A)

$\operatorname{cof}(A)$ = cofactor matrix of A $= (C_{ij})$

$\rightarrow C_{ij} = (-1)^{i+j} \det(A \text{ with } i^{th} \text{ row } \& \ j^{th} \text{ col removed})$

Check $A \cdot \operatorname{cof}(A)^T = \det(A)\, I$

$$Au \times Av = \operatorname{adj}(A)^T \, u \times v$$

Stokes thm Pf: $(\Sigma, \hat{n})$ oriented surface (bdd)

$F : \Sigma \longrightarrow \mathbb{R}^3$ $\quad C^1$

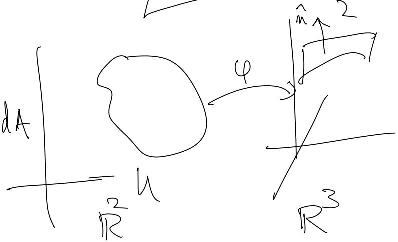
$$\oint_{\partial \Sigma} F \cdot dl = \int_{\Sigma} \nabla \times F \cdot \hat{n} \, dS$$

$$\int_U \nabla \cdot F \, dV = \int_{\partial U} F \cdot \hat{n} \, dS$$

$$\int_a^b f' \, dx = f(b) - f(a)$$

Pf: (1) Param Σ : $U \subseteq \mathbb{R}^2$

(a) $$\int_{\Sigma} \nabla \times F \cdot \hat{n} \, dS = \int_U (\nabla \times F) \circ \varphi \cdot (\partial_1 \varphi \times \partial_2 \varphi) \, dA$$



(2) $$\int_{\partial\Sigma} F \cdot dl = \int_{\partial U} (D\varphi)^T F \circ \varphi \cdot dl = \int_{\partial U} G \cdot dl$$

$$\overset{\text{Greens thm}}{=} \int_U (\partial_1 G_2 - \partial_2 G_1) \, dA$$

$$= \int_U (\nabla \times F \circ \varphi) \cdot (\partial_1 \varphi \times \partial_2 \varphi) \, dA$$

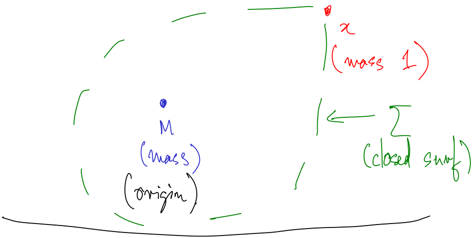
$$= \int_\Sigma F \cdot \hat{n} \, dS$$

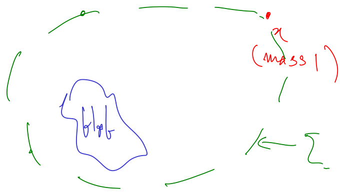
QED.

$$G = (D\varphi)^T F \circ \varphi$$

$$\partial_1 G_2 - \partial_2 G_1 = (\partial_1 \varphi \times \partial_2 \varphi) \cdot (\nabla \times F) \circ \varphi$$

Gauss law: $\mathbb{R}^3$





Q: $g(x)$ = Force exp by a unit mass at x.

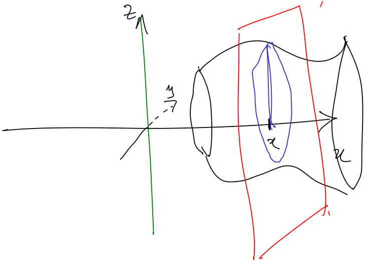
$$g(x) = -GM \frac{x}{|x|^3}$$

∇N (Newton potential)

$g(x)$ = force exp by a unit mass at x

$$\int_\Sigma g(x)\cdot \hat{n}\, dS = (\text{const})\,(\text{mass enclosed by } \Sigma)$$

Q 2



$$\Sigma = \left\{ y^2 + z^2 = f(x)^2 \right\}$$