Q 3:

$\partial_1 G_2 - \partial_2 G_1$

$$G = (D\varphi)^T F \circ \varphi$$

$$G_1 = \sum \partial_1 \varphi_j F_j \circ \varphi$$

$$\boxed{G_2 = \sum_j \partial_2 \varphi_j F_j \circ \varphi}$$

$$\partial_1 (F_j \circ \varphi) = \partial_1 \cancel{F_j \circ \varphi \partial_1 \varphi}$$

$$= \sum (\partial_k F_j) \circ \varphi \, \partial_1 \varphi_k$$

$$\partial_t F(x,y) = \partial_x F \partial_t x + \partial_y F \partial_t y$$

$$\partial_1 G_2 = \sum_j \partial_1 \partial_2 \varphi_j F_j \circ \cancel{\varphi} + \sum_{j,k} \partial_2 \varphi_j \partial_k F_j \partial_1 \varphi_k$$

$$\partial_2 G_1 = \sum_j \partial_2 \partial_1 \varphi_j + \sum_{j,k} \partial_1 \varphi_j \partial_k F_j \partial_2 \varphi_k$$

$$\Rightarrow \partial_1 G_2 - \partial_2 G_1 = \sum_{jk} \underbrace{\left(\partial_2 \varphi_j \partial_1 \varphi_k - \partial_1 \varphi_j \partial_2 \varphi_k \right)}_{} \partial_k F_j$$

$$\det\begin{pmatrix} \partial_1 \varphi_k & \partial_2 \varphi_k \\ \partial_1 \varphi_j & \partial_2 \varphi_j \end{pmatrix}$$

$$\Rightarrow \partial_1 G_2 - \partial_2 G_1 = \det\begin{pmatrix} \partial_1 \varphi_2 & \partial_2 \varphi_2 \\ \partial_1 \varphi_1 & \partial_2 \varphi_1 \end{pmatrix} \partial_2 F_1 + \underbrace{\det\begin{pmatrix} \partial_1 \varphi_1 & \partial_2 \varphi_1 \\ \partial_1 \varphi_2 & \partial_2 \varphi_2 \end{pmatrix}} \partial_1 F_2$$

$(j=1, k=2)$

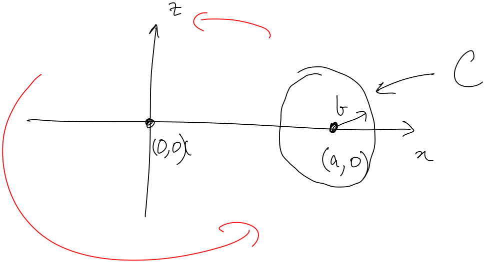
$$= -\det(D\varphi)\, \partial_2 F_1 + \det(D\varphi)\, \partial_1 F_2$$

$$= \det(D\varphi)\left(\partial_1 F_2 - \partial_2 F_1\right) \longleftarrow$$

composed with φ but I didn't write.

Q5: $a > b > 0$

Γ = Torus



① Param surface

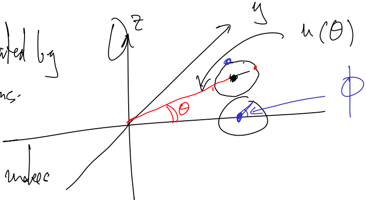
② Compute the S-int.

$\Theta \longrightarrow$ angle the circle C is rotated by

$\phi \longrightarrow$ Consider the cross section of torus.

in the z & $u(\Theta)$

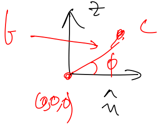
ϕ = angle pt on the torus makes to $u(\Theta)$.



Formula: (x, y, z) on the torus. want x, y, z in terms of θ & ϕ

① $u(\theta)$ = center of rotated circle

$= (a\cos\theta, a\sin\theta, 0)$



$\hat{n}(\theta)$ = unit vector in direction u

$= (\cos\theta, \sin\theta, 0)$

② c = pt in the z - $\hat{n}$ plane that makes an angle ϕ to the $\hat{n}$ axis & is a dist $\underline{b}$ away from 0

$$= b\left(\sin\phi \; e_3 + \cos\phi \; \hat{n}\right) \qquad \left(e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right)$$

(2') $(x, y, z) =$ pt c shifted by $u(\theta)$

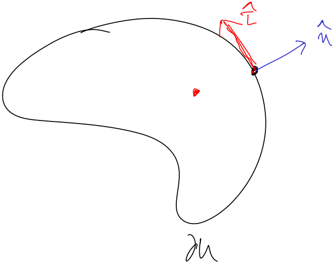
$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = c + u(\theta) = b \sin\phi \, e_3 + b\cos\phi \, \hat{n}(\theta) + u(\theta)$$

$$= b\sin\phi \, e_3 + b\cos\phi \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} + \begin{pmatrix} a\cos\theta \\ a\sin\theta \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} a\cos\theta + b\cos\phi\cos\theta \\ a\sin\theta + b\cos\phi\sin\theta \\ b\sin\phi \end{pmatrix}$$

$\boxed{4a}$

$$\int_{\partial U} F \cdot \hat{n} \; |d\ell| = \int_U \nabla \cdot F \; dA$$



$\hookrightarrow$ Fiona: $G = \begin{pmatrix} +F_2 \\ -F_1 \end{pmatrix}$

$\hat{n} = \begin{pmatrix} \hat{n}_1 \\ \hat{n}_2 \end{pmatrix}$

$\hat{t}$ = unit vector tgt to ∂U (CCW)

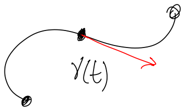
$\hat{t} = \begin{pmatrix} -\hat{n}_2 \\ \hat{n}_1 \end{pmatrix}$

$$\int_{\partial U} F \cdot \hat{n} \; |dl| = \int_{\partial U} \begin{pmatrix} +G_2 \\ -G_1 \end{pmatrix} \cdot \begin{pmatrix} \hat{n}_1 \\ \hat{n}_2 \end{pmatrix} |dl|$$

$$= \int_{\partial U} \begin{pmatrix} G_1 \\ G_2 \end{pmatrix} \cdot \underbrace{\begin{pmatrix} -\hat{n}_2 \\ \hat{n}_1 \end{pmatrix}}_{\hat{\tau}} |dl| = \int_{\partial U} G \cdot \hat{\tau} \; |dl|$$

$$\overset{\shortparallel}{\int_{\partial U}} G \cdot dl = \text{Green's thm.}$$

<u>Claim:</u> $\displaystyle\int_{\partial U} G \cdot \hat{\tau} \; |dl| = \int_{\partial U} G \cdot dl$



Check: γ be a param.

Q: $\hat{\tau} = \dfrac{\gamma'(t)}{|\gamma'(t)|}$

At $\gamma(t)$, $\gamma'(t)$ is tgt to Γ

$\Rightarrow$ unit tgt vec: $\gamma'(t)/|\gamma'(t)|$

$$\Rightarrow \int_{\partial U} G \cdot \hat{t} \; |dl| = \int_{\partial U} G\circ\gamma \cdot \frac{\gamma'(t)}{\cancel{|\gamma'(t)|}} \; \cancel{|\gamma'(t)|} \; dt$$

$$= \int_{\partial U} G\circ\gamma \cdot \gamma'(t) \; dt = \int_{\partial U} G \cdot dl$$