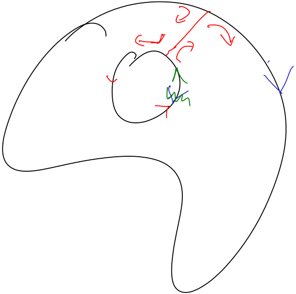
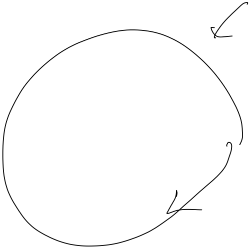
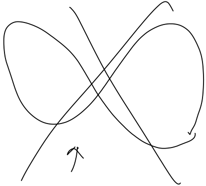
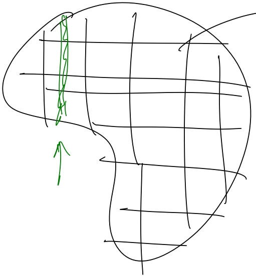






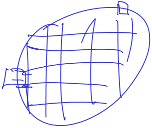


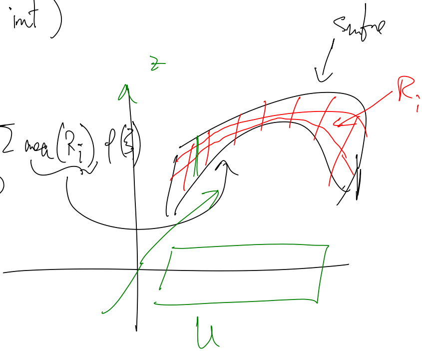
$$\lim_{\|P\|\to 0} \sum \text{area}(R_i)\, \rho(\xi_i)$$

(Area int)

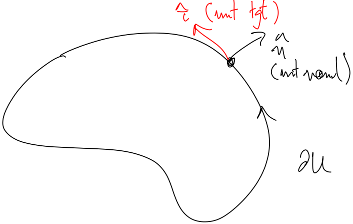
Surface int: $\displaystyle \lim_{\|P\|\to 0} \sum \text{area}(R_i)\, \rho(\xi_i)$

$U \subseteq \mathbb{R}^2$





Q4 $\quad U \subseteq \mathbb{R}^2$



$$\boxed{\int_{\partial U} F \cdot \hat{n}\, |dl| = \int_U \underbrace{(\nabla \cdot F)}_{\text{div.}}}$$

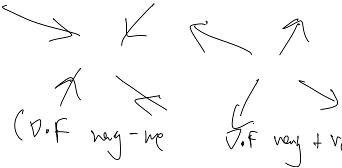
$F : U \to \mathbb{R}^2$

$\nabla \cdot F = \partial_1 F_1 + \partial_2 F_2$.

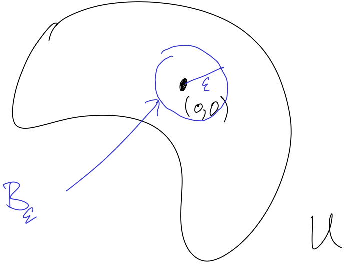
$\left(\forall F : \mathbb{R}^d \to \mathbb{R}^d, \quad \nabla \cdot F = \sum_{i=1}^{d} \partial_i F_i \right)$

Hint: Find a fn $\boxed{G} : U \to \mathbb{R}^2$

so that $\displaystyle\int_{\partial U} F \cdot \hat{n}\, |dl| = \oint_{\partial U} G \cdot dl$



4b



$$P = \frac{1}{2\pi}\,\frac{-y}{x^2+y^2}$$

$$Q = \frac{1}{2\pi}\,\frac{x}{x^2+y^2}$$

$$W(\Gamma) = \oint_\Gamma P\,dx + Q\,dy$$

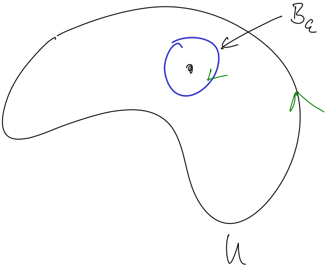
Defined as long as $0 \notin \Gamma$

$W(\partial U) \longrightarrow$ defined as long as $0 \notin \partial U$

needs clever idea $\longleftarrow$ Case I: $0 \in U$

Case II: $0 \notin U$ (Green's thm: $\int_{\partial U} P\,dx + Q\,dy = \int_U \underbrace{\partial_x Q - \partial_y P}_{= 0}$)

Green's thm WORKS in $U - B(0,\varepsilon)$



① $V_\varepsilon = U - B(0,\varepsilon)$

② Green $\displaystyle\oint_{\partial V_\varepsilon} P\,dx + Q\,dy = \int_{V_\varepsilon} (\quad) = 0$

$$0 = \oint_{\partial V_\varepsilon} P\,dx + Q\,dy = \underbrace{\oint_{\partial U}}_{\substack{(ccw) \\ W(\partial U)}} + \underbrace{\int_{\partial B(0,\varepsilon)}}_{\substack{(cw) \\ -1}}$$

Q3: $G = (D\varphi)^T F \circ \varphi$

$$(D\varphi)_{ij} = \left(\partial_j \varphi_i\right) \qquad \underbrace{\left(\partial_i \varphi_j\right)_{ij}}\ \underbrace{\left(F_i\right)_i}$$

$$F = \left(F_i\right)$$

G_i = i^{th} row of $(D\varphi)^T$ times $F \circ \varphi$

$$\sum_j \partial_i \varphi_j \, F_j \circ \varphi \qquad \begin{pmatrix} \partial_i \varphi_1 & \partial_i \varphi_2 \end{pmatrix} \begin{pmatrix} F_1 \circ \varphi \\ F_2 \circ \varphi \end{pmatrix}$$

$$G_1 = \sum_j \partial_1 \varphi_j \, F_j \circ \varphi$$

$$\boxed{\partial_t F(x,y) = \partial_x F \frac{dx}{dt} + \partial_y F \frac{dy}{dt}}$$

$$\partial_2 G_1 = \sum_j \partial_2 \left(\partial_1 \varphi_j \, F_j \circ \varphi \right)$$

$$\sum_j \left(\partial_1 \partial_2 \varphi_j \, F_j \circ \varphi + \partial_1 \varphi_j \, \underbrace{\partial_2 \left(F_j \circ \varphi \right)}_{\sum_k \partial_k F_j \circ \varphi \, \partial_2 \varphi_k} \right)$$

$$\partial_1 G_2 = \cancel{*}$$