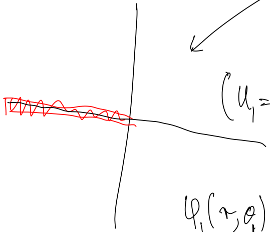
2 Polar coordinates:

1



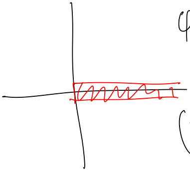
$$\varphi_1 : \mathbb{R}^+ \times (-\pi, \pi) \longrightarrow U_1$$

$$\left(U_1 = \mathbb{R}^2 - \{-\text{ve } x \text{ axis}\} \right)$$

$$\varphi_1(r, \theta_1) = \begin{pmatrix} r \cos \theta_1 \\ r \sin \theta_1 \end{pmatrix}$$

Compute $\nabla \theta_1 \longrightarrow \frac{1}{x^2+y^2} \begin{pmatrix} -y \\ x \end{pmatrix}$

2

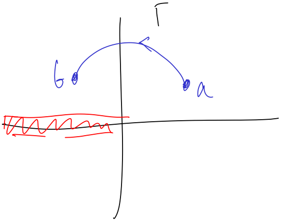


$$\varphi_2 : \mathbb{R}^+ \times (0, 2\pi) \rightarrow U_2$$

$$\left(U_2 = \mathbb{R}^2 - \{+\text{ve } x \text{ axis}\} \right)$$

$$\varphi_2(r, \theta_2) = \begin{pmatrix} r \cos \theta_2 \\ r \sin \theta_2 \end{pmatrix}$$

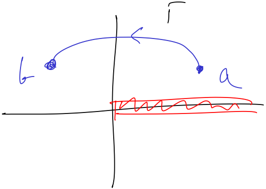
$\nabla \theta_2 \xrightarrow{\text{Compute}} \frac{1}{x^2+y^2} \begin{pmatrix} -y \\ x \end{pmatrix}$



Compute $\frac{1}{2\pi}\int_\Gamma \frac{1}{x^2+y^2}\left(-y\,dx + x\,dy\right)$

$= \frac{1}{2\pi}\int_\Gamma \nabla\theta_1 \cdot dl$

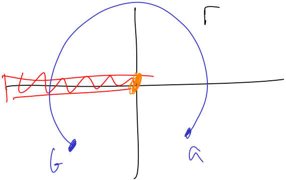
$= \frac{1}{2\pi}\left(\theta_1(b) - \theta_1(a)\right)$



Compute $\frac{1}{2\pi}\int_\Gamma \frac{1}{x^2+y^2}\left(-y\,dx + x\,dy\right)$

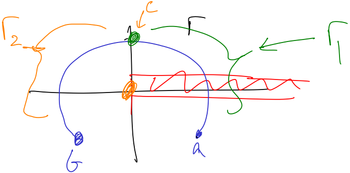
$= \frac{1}{2\pi}\int_\Gamma \nabla\theta_2 \cdot dl$

$= \frac{1}{2\pi}\left(\theta_2(b) - \theta_2(a)\right)$



Q: Compute $\frac{1}{2\pi}\int_\Gamma \frac{1}{x^2+y^2}\begin{pmatrix}-y\\ x\end{pmatrix}\cdot d\ell$

$= \cancel{\frac{1}{2\pi}\int_\Gamma \nabla\theta_1 \cdot d\ell}$ (Won't work. for this to work need $\Gamma \subseteq U_1$)



Compute $\frac{1}{2\pi}\int_\Gamma \frac{1}{x^2+y^2}\begin{pmatrix}-y\\ x\end{pmatrix}\cdot d\ell$

$\neq \int_\Gamma \nabla\theta_2 \cdot d\ell$ (need $\Gamma \subseteq U_2$ for this to work)

Split Γ into $\boxed{\Gamma_1 \cup \Gamma_2}$

Obs: $\Gamma = \Gamma_1 \cup \Gamma_2$ $\quad$ ① $\Gamma_1 \subseteq U_1$ & ② $\Gamma_2 \subseteq U_2$

$$\frac{1}{2\pi}\int_{\Gamma} \underbrace{\frac{-y\,dx + x\,dy}{x^2+y^2}}_{F} = \frac{1}{2\pi}\int_{\Gamma} F\cdot dl = \underbrace{\frac{1}{2\pi}\int_{\Gamma_1} F\cdot dl}_{} + \underbrace{\frac{1}{2\pi}\int_{\Gamma_2} F\cdot dl}_{}$$

$$\frac{1}{2\pi}\int_{\Gamma_1} \nabla\theta_1 \cdot dl + \frac{1}{2\pi}\int_{\Gamma_2} \nabla\theta_2 \cdot dl$$

$$\frac{1}{2\pi}\left(\underbrace{\theta_1(c)-\theta_1(a)}\right) + \frac{1}{2\pi}\left(\underbrace{\theta_2(b) - \theta_2(c)}\right)$$

$$= \frac{1}{2\pi}\left(\theta_2(b) - \theta_1(a) + \left(\theta_1(c) - \theta_2(c)\right) \right)$$

Say Γ_2 starts here.

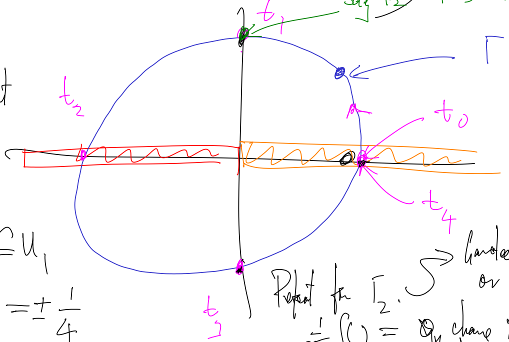
Split $\Gamma = \bigcup \Gamma_i$

Each Γ_i is completely cont in either U_1

or comp cont in U_2

Γ_1 = part between t_0 & t_1 $\subseteq U_1$

$$\frac{1}{2\pi}\int_{\Gamma_1} (\ ) = \frac{1}{2\pi}\int_{\Gamma_1} \nabla\theta_1 \cdot d\ell = \pm\frac{1}{4}$$



Repeat for Γ_2. Guaranteed $\Gamma_2 \subseteq U_1$ or $\Gamma_2 \subseteq U_2$

$$\frac{1}{2\pi}\int (\ ) = \text{change in } \theta_1 \text{ OR change in } \theta_2 = \pm \frac{1}{4}$$

Divide Γ into $\Gamma_1, \dots \Gamma_n$

(Γ_i is the piece of the curve between t_i & t_{i+1})

<u>Claim</u> $\exists k \in \{1,2\}$ $\ni$ $\Gamma_i \subseteq U_k$

$$\Rightarrow \frac{1}{2\pi}\int_{\Gamma_i} (\ ) \, dl = \frac{1}{2\pi}\int_{\Gamma_i} \nabla \theta_k \cdot dl \; \begin{cases} \to +\frac{1}{4} \\ \to -\frac{1}{4} \end{cases}$$

Q 2 (last time) (IOU Email with orientation)

$(-1, -1, -1)$
$(1, 1, 1)$
$(-1, 1, -1)$
$(1, -1, 1)$

$(-1, -1, -1)$
$(-1, 1, -1)$
$(1, 1, 1)$
$(1, -1, 1)$