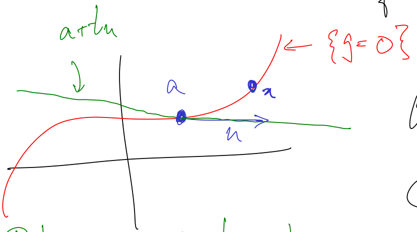
HW 10 Q 2b: $u \in$ Tgt space $\Rightarrow (Hf_a\, u)\cdot u \geq 0$

Constr. $\{g=0\}$. Min f subj to $g=0$

Q: $u \in$ Tgt space of $\{g=0\}$ at a & f has a constr min at a $\}$?$\Rightarrow$



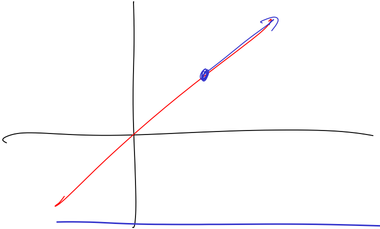
Pant cos g is linear.!

Know f has a const min at a.

Guessed $(Hf_a\, u)\cdot u \geq 0$

$\hookrightarrow$ f has a local min in the direction u?

$\hookrightarrow$ $f(a+tu)$ has a local min at 0



① $D g_a\, u = 0$

② $D f_a\, u = 0$ (Lagrange)

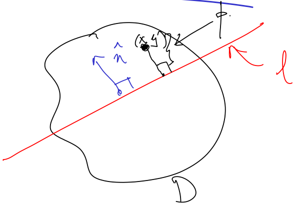
$$f(x) = f(a) + Df_a(x-a) + \frac{1}{2}\left(Hf_a(x-a)\right)\cdot(x-a)$$

HW 11: Q 3: $D \subseteq \mathbb{R}^2$

$\rho(x,y) \to$ density

$l \to$ straight line. Balance D on l.

$$\text{Torque} = T_l = \int_D \rho\, \underset{\uparrow\ \perp \text{ dist}}{d}\, dA$$

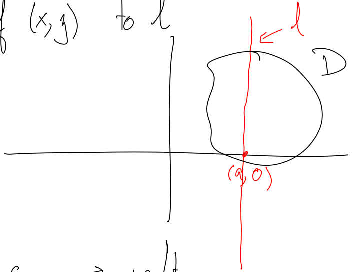


$\underset{\sim}{d} = \underset{\sim}{p} \cdot \hat{n}$, = perp distance of (x, y) to ℓ

(a) Given $a \in \mathbb{R}$, $\ell =$

$T_\ell = 0 \iff a = \frac{1}{M} \int_D x \, \rho \, dA$,

$M = \int_D \rho \, dA$ = total mass.



(Aside $\to$ weighted averages: $c_1, \cdots c_n \to$ weights.

$x_1, \cdots x_n \to$ values.

weight av of x_i with weights c_i: $\dfrac{\sum c_i x_i}{\sum c_i}$

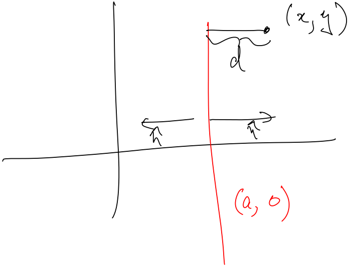
(think of $\sum c_i = 1$))

Think of $\frac{1}{M}\int_D x\,\rho \, dA$ ~~at~~ as the "average x-coordinate in D"

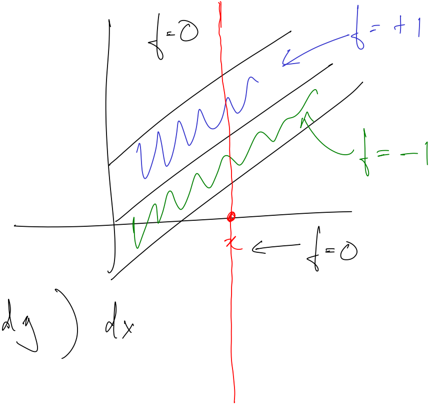
Try: $T_\ell = \int_D \rho \cdot d \, dA$

$$0 = \int_D \rho(x,y)\,(x-a)\,dx\,dy$$

$$\iff a\underbrace{\int_D \rho \, dA}_{M} = \int_D x\,\rho(x,y)\,dx\,dy$$



Fubini Eg from class:
Check that conditions for Fubini don't apply



Compute $\displaystyle\int_{x\in\mathbb{R}} \int_{y\in\mathbb{R}} |f(x,y)|\, dy \; dx$

$$= \int_{x\in\mathbb{R}} \left(\int_{y\in\mathbb{R}} \mathbb{1}_{\{\text{in blue} \cup \text{green}\}} \, dy \right) dx$$

$$= \int_{x\geq 0} (2)\, dx = +\infty \, !$$

f = horrible formula.

$\hookrightarrow$ To check Fubini Only NTS $\displaystyle\int_{x\in R}\int_{y\in S_x} |f(x,y)|\,dy\,dx < \infty$

80% of the time: f is bounded (i.e. $|f(x,y)| \leq C$ for some $C \in \mathbb{R}$)

$$\Rightarrow \int_{x\in R}\int_{y\in S_x} |f(x,y)|\,dy\,dx \leq \underbrace{\int_{x\in R}\int_{y\in S_x} C\,dy\,dx}_{C\ \cdot\ \mathrm{Area}(W)} < \infty.$$

(80% of the time)

$g : \mathbb{R}^m \to \mathbb{R}^n$ is linear

$Dg_a = g$

$g(x) = Mx$ ($M \to$ is a matrix).

$a \in \mathbb{R}^m$. $Dg_a = M$ (not Ma)