Midterm 2 Q 3: $S = \{2xy + z^2 = -1\}$ $a = (-1, 1, 1)$

Let U = Tgt space. Want u & v, $\underbrace{u \cdot v = \frac{1}{2}|u|\,|v|}$.

$|u|\,|v| \cos\theta$ ✓

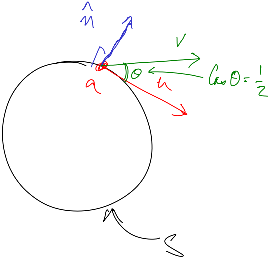
Q: ① Find 1 tgt vector.

$\rightarrow \operatorname{Ker}(Df|_a) = \operatorname{Ker}\begin{pmatrix} 2y & 2x & 2z \end{pmatrix}_a$

$= \operatorname{Ker}\left(\begin{pmatrix} 1 & -1 & 1 \end{pmatrix}\right)$

Pick $u = (1, 1, 0)$

② Normal vector: $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

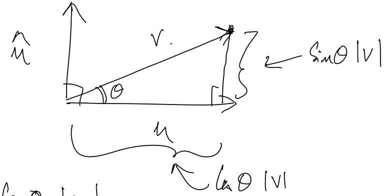


Given θ, & $\hat{n} \ni \hat{n} \cdot u = 0$

Find $v \ni u \cdot v = |u|\,|v| \cos\theta$

Say $|u| = 1$ & $|\hat{n}| = 1$

$\cos\theta = \frac{|u|}{|v|} \Rightarrow |v| = \cos\theta\, |u|$



$v = u \cos\theta\, \cancel{|v|} + \hat{n} \sin\theta\, \cancel{|v|}$

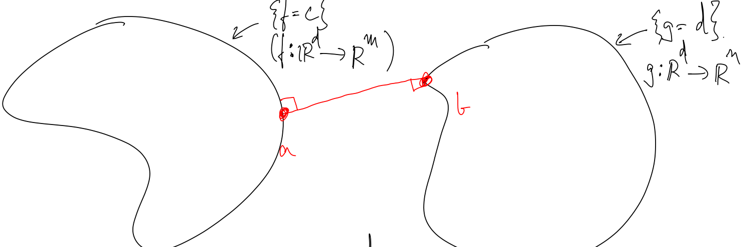
$$\boxed{v = \cos\theta\, u + \sin\theta\, \hat{n}}$$

Check: $u \cdot v = |u|\,|v| \cos\theta$?

① $|v|^2 = v \cdot v = \cos^2\theta + \sin^2\theta = 1$

② $u \cdot v = \cos\theta$ ✓

HW 10 Q 3:



Q: Say dist between M & N M $(M, N \subseteq \mathbb{R}^d)$ N
is min at $a \in M$ & b in N.

Is $a - b \perp$ tgt spce M at a.
& " " " " N at b } → Yes: Use Lag & minimise $|x-y|^2$ subj to $f(x) = c$ & $g(y) = d$

Let $(x,y) \in \mathbb{R}^{2d}$ Let $h: \mathbb{R}^{2d} \longrightarrow \mathbb{R}$ be $h(x,y) = |x-y|^2$

Let $F(x,y) = \begin{pmatrix} f(x) \\ g(y) \end{pmatrix}$. $F: \mathbb{R}^{2d} \longrightarrow \mathbb{R}^{m+n}$

Min h subj to the constr $F = \begin{pmatrix} c \\ d \end{pmatrix}$

↳ Use Lagrange & get exactly $a-b \perp$ Tgt space of Mat a.

At constr min/max (a,b): Lag $\to \exists \lambda_i$ + $\nabla h(a,b) = \sum \lambda_i \nabla f_i$

$h(x,y) = |x-y|^2 = \sum (x_i - y_i)^2$. $\nabla h = \begin{pmatrix} \partial_{x_1} h \\ \vdots \\ \partial_{x_d} h \\ \partial_{y_1} h \\ \vdots \\ \partial_{y_d} h \end{pmatrix}$

$$\Rightarrow \nabla h(x,y) = \begin{pmatrix} 2(x_1 - y_1) \\ \vdots \\ 2(x_d - y_d) \\ 2(y_1 - x_1) \\ \vdots \\ 2(y_d - x_d) \end{pmatrix} = 2 \begin{pmatrix} x - y \\ y - x \end{pmatrix}$$

(2) Compute ∇F_i : $F(x,y) = \begin{pmatrix} f(x) \\ g(y) \end{pmatrix}$:

(1) $i \leq m$: $\nabla F_i = \begin{pmatrix} \nabla_x f_i \\ 0 \end{pmatrix}$

(2) $i > m$: $\nabla F_i = \begin{pmatrix} 0 \\ \nabla_y g_{i-m} \end{pmatrix}$

At constr min/max: $\nabla h(a,b) = \sum \lambda_i \nabla F(a,b)$

$$2\begin{pmatrix} a-b \\ b-a \end{pmatrix} = \sum_{i=1}^{m} \lambda_i \begin{pmatrix} \nabla f_i(a) \\ 0 \end{pmatrix} + \sum_{i=1}^{n} \lambda_{i+m} \begin{pmatrix} 0 \\ \nabla g_i(b) \end{pmatrix}$$

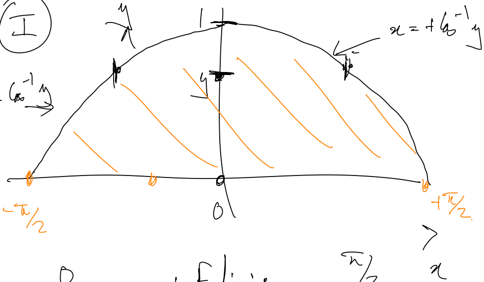
$$a-b = \frac{1}{2} \sum \lambda_i \underbrace{\nabla f_i(a)}$$

normal to tgt space of M at a.

$\Rightarrow a-b$ is normal to the tgt space of M at a.

Q5b

$$\int_{y=0}^{1}\int_{x=-\cos^{-1}y}^{+\cos^{-1}y} e^{\sin x}\,dx\,dy \leftarrow \text{I}$$

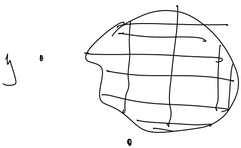


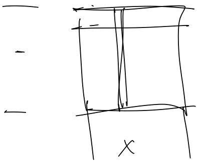
$$\int_{x=-\pi/2}^{\pi/2}\int_{y=0}^{\cos x} e^{\sin x}\,dy\,dx$$

$-\pi/2$ $\quad$ 0

$\pi/2$

Fubini: $\text{I} = \int_{x=\_}^{\_}\int_{y=\_}^{\_}$





For Fubini need either (1) $\displaystyle\int_{y\in\mathbb{R}}\int_{x\in T_y} |f(x,y)|\,dx\,dy < \infty$

OR (2) $\displaystyle\int_{x\in\mathbb{R}}\int_{y\in S_x} |f(x,y)|\,dy\,dx < \infty$

Say $f>0$: Then ~~Just~~ Just compute either (1) or (2)

if the answer is $<\infty$, then Fubini applies.

Asked to compute (1). Instead compute (2). If the ans is $<\infty$ then ~~Fub~~ Fubini applies & (2) $=$ (1) !