$f \qquad Df_a = \begin{pmatrix} 2 & 6 & 8 \\ 2 & 6 & 9 \end{pmatrix}$

$f(g_1(t), \cancel{*}\, 2+t, g_2(t)) = 0$ near $g(0) = (0, 1)$.

Compute $g_1''(0)$ & $g_2''(0)$

Let $h(t) = \begin{pmatrix} g_1(t) \\ 2+t \\ g_2(t) \end{pmatrix}$ $\qquad f \circ h(t) = 0$

$Df_{h(t)} \cdot h'(t) = 0$

$\begin{pmatrix} 2 & 6 & 8 \\ 2 & 6 & 9 \end{pmatrix} h'(t) = 0$

diff again $\cancel{\begin{pmatrix} 2 & 6 & 8 \\ 2 & 6 & 9 \end{pmatrix} h''(t)} = 0$ & ~~solve (Wrong !!!)~~

Wrong

$$Df_{h(t)} \cdot h'(t) = 0 \longrightarrow \sum_j \partial_j f_i(h(t))\, h_j'(t) = 0 \quad (i\text{th coordinate})$$

diff again

$$\underbrace{\sum_{jk} \partial_k \partial_j f_i \; h_j'(t)\, h_k'(t)}_{\left(\left(Hf_i\right)_{h(t)} h'(t)\right) \cdot h'(t)} + \underbrace{\sum_j \partial_j f_i(h(t))\, h_j''(t)}_{\left(Df_{h(t)} h''(t)\right)_i}$$

① $f \circ h(t) = 0 \longrightarrow \text{Diff}$ ② $Df_{h(t)}\, h'(t) = 0 \longrightarrow \text{Diff.}$

③ $$Df_{h(t)}\, h''(t) + \begin{pmatrix} (Hf_1)_{h(t)} h'(t) \cdot h'(t) \\ (Hf_2)_{h(t)} h'(t) \cdot h'(t) \end{pmatrix} = 0$$

$$\boxed{h(t) = \begin{pmatrix} g_1(t) \\ 2+t \\ g_2(t) \end{pmatrix} \qquad h'(0) = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}}$$

Solve: ① $$\begin{pmatrix} 2 & 6 & 8 \\ 2 & 6 & 9 \end{pmatrix} \begin{pmatrix} g_1'(0) \\ 1 \\ g_2'(0) \end{pmatrix} = 0 \iff \begin{pmatrix} 2 & 8 \\ 2 & 9 \end{pmatrix} g'(0) = \begin{pmatrix} -6 \\ -6 \end{pmatrix}$$

(solve for g'). $\to g'(0) = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$

② g'': $$\begin{pmatrix} 2 & 6 & 8 \\ 2 & 6 & 9 \end{pmatrix} \begin{pmatrix} g_1''(0) \\ 0 \\ g_2''(0) \end{pmatrix} + \begin{pmatrix} ((Hf_1)_a\, h'(0)) \cdot h'(0) \\ ((Hf_2)_a\, h'(0)) \cdot h'(0) \end{pmatrix} = 0$$

& solve.

5a) $\displaystyle\int_{x=-1}^{1}\left(\int_{y=1}^{2} \sin(xy^2)\, dy\right) dx$

$\sum |a_i| < \infty$

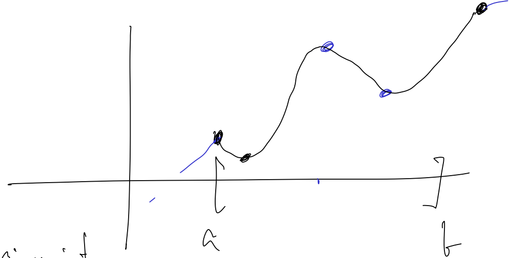
$\Rightarrow \sum$ (any re-arrangement) = all equal

Compute $\int \sin(y^2)\, dy \longrightarrow$ not poss.

(Fubini) $\displaystyle\int_{y=1}^{2}\left(\int_{x=-1}^{1} \sin(xy^2)\, dx\right) dy$

1D calculus:

max/min f on $[a, b]$.



① Solve for $f' = 0$ (gives int min/max).

② Endpoint min/max: ⓐ Left endpoint: $f'(a) > 0 \Rightarrow$ local endpt min

$f'(a) < 0 \Rightarrow$ " " max

ⓑ Right endpt $\rightarrow$ flip signs.

2D Version :

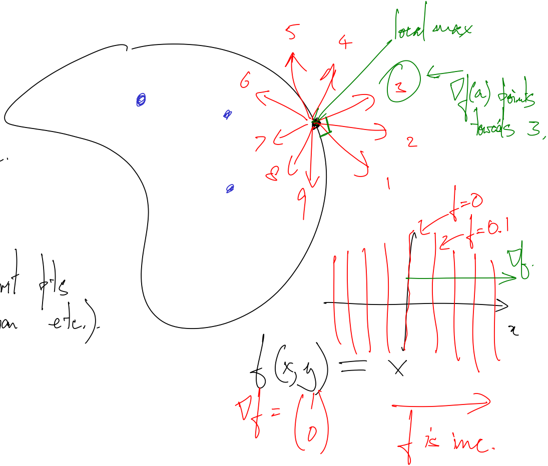
max/min on some closed set C.

$U \to$ interior region

$\partial U \to$ boundary.

① Int min/max : Find crit pts (Hessian etc.)

② Boundary



③ $M = \{f = c\}$

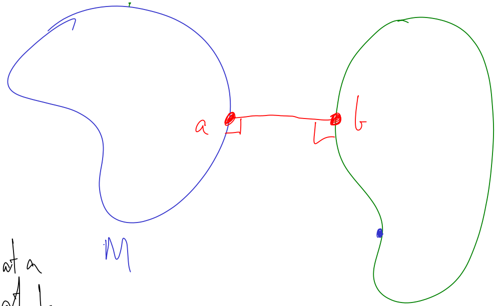
$N = \{g = d\}$

min dist at $a \in M$

$b \in N$.

Q: $a - b \perp$ tgt space M at a

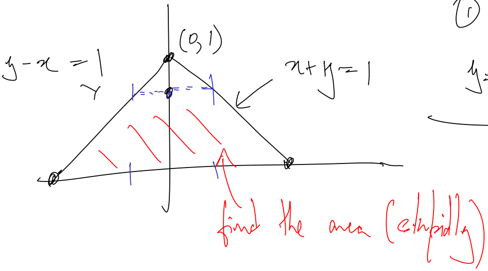
$\perp$ " " N at b.



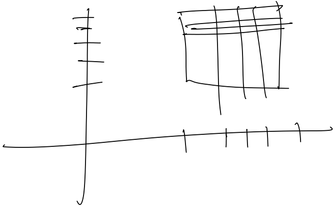
Try $h(x,y) = |x-y|^2$. Min f subject to ~~$x \in M$~~ ~~& $y \in N$~~ $f(x) = c$, $g(y) = d$.

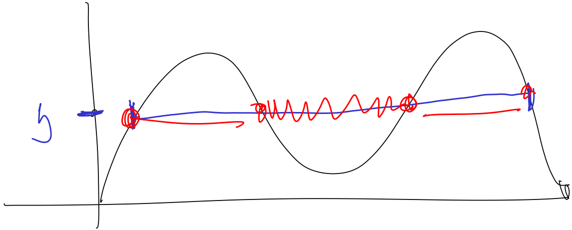
Min $h(x,y)$ subj to the constr $G(x,y) = \begin{pmatrix} c \\ d \end{pmatrix}$ where $G(x,y) = \begin{pmatrix} f(x) \\ g(y) \end{pmatrix}$

Constr: $g : \mathbb{R}^{d+n} \longrightarrow \mathbb{R}^n$ $g(x) = 0$
Constr linear means $g : \mathbb{R}^{d+n} \longrightarrow \mathbb{R}^n$ is linear.



(1) $\int_{y=0}^{1} \int_{x=(y-1)}^{(1-y)} 1 \, dx \, dy$




b