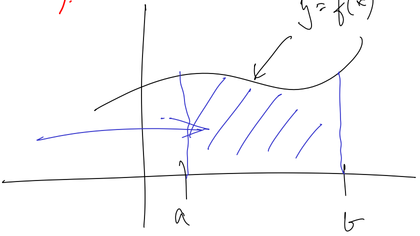
① I forgot to record office hours today (sorry!).
(PDF of notes avail).

② Midterm: (Ends at hw until). (Good luck!!)

Integration: (Not on Midterm!!)

1 var integration:

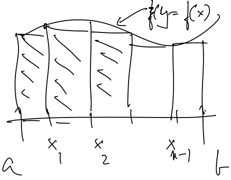
area under curve $= \int_a^b f(x)\,dx$



Riemann integrals: $P = \{ a = x_0 < x_1 \cdots x_n = b \}$.

norm $\longrightarrow \|P\| = \max\limits_{i \le n} x_{i+1} - x_i$

Approximate area under the curve by $= \boxed{\sum_{i=0}^{n-1} f(x_i)(x_{i+1} - x_i)}$



R-int: $\displaystyle\int_a^b f(x)\,dx \overset{\text{def}}{=} \lim_{\|P\| \to 0} \sum_{i=0}^{n-1} f(x_i)(x_{i+1} - x_i)$

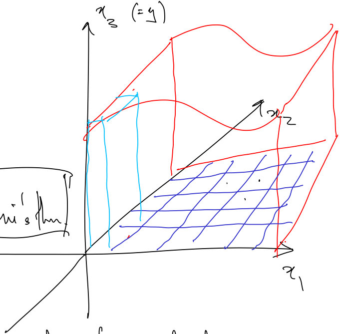
Goal now: Higher dim version.

Surface: $\{x_3 = f(x_1, x_2) \mid x_1, x_2 \in [0,1]^2\}$

Volume under the graph?

(1) Integrate x_1 first & then x_2.
(" x_2 " & then x_1)

Fubini's thm



(2) (Basic defn) Approximate the volume under the surface by cuboids.

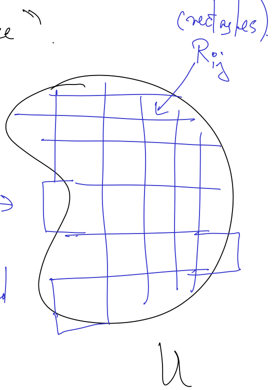
Say $U \subseteq \mathbb{R}^2$ is open (bdd) "nice".

Let P be a "partition" of U into rectangles.

$$P = \bigcup_{i,j} R_{ij}$$

$R_{ij} \to$ rectangles which cover U

ignore all rectangles that are not completely contained in U.



$\forall R_{ij}$ let ξ_{ij} be any point in R_{ij}.

Approximate vol under the surface $S = \{x_3 = f(x_1, x_2) \mid (x_1, x_2) \in U\}$.

by $\sum_{i,j} \underbrace{\text{area}(R_{ij})}_{\text{base}} \cdot \underbrace{f(\xi_{ij})}_{\text{height}}$.

notation

area int.

Riemann integral: Define $\int_U f \, dx_1 dx_2 = \iint_U f \, dx_1 \, dx = \iint_U f \, \overbrace{dA}$

(Us: usually write $\int_U f \, dx_1 dx_2$ or $\int_U f \, dA$)

$$\lim_{\|P\| \to 0} \sum_{i,j} \text{area}(R_{ij}) f(\xi_{ij})$$

(Q: What is $\|P\|$?)

~~Try $\|P\| = \max \text{area}(R_{ij})$?~~

$\|P\| = \max$ side length of R_{ij}

Computing integrals: 1D $\longrightarrow$ Use anti-derivatives (FTC).

2D: $\longrightarrow$ Reduce to 1D. $\longrightarrow$ Doesn't always work.

Say $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$. Compute $\int_{\mathbb{R}^2} f \, dx_1 \, dx_2$

Guess: Compute $$\int_{x_1=-\infty}^{\infty} \left(\int_{x_2=-\infty}^{\infty} f(x_1, x_2) \, dx_2 \right) dx_1 = \int_{\mathbb{R}^2} f \, dx_1 \, dx_2.$$

also do $\displaystyle\int_{x_2=-\infty}^{\infty}\left(\int_{x_1=-\infty}^{\infty} f(x_1, x_2)\, dx_1\right) dx_2$

Doesn't always work. (Fubini's theorem gives condition on when it does (Friday)).

$f: \mathbb{R}^{d+n} \longrightarrow \mathbb{R}$

$Df_a : \mathbb{R}^{d+n} \longrightarrow \mathbb{R}$

$Dg_a = \begin{pmatrix} \leftarrow \nabla g_1(a) \rightarrow \\ \vdots \\ \leftarrow \nabla g_n(a) \rightarrow \end{pmatrix}$

$\text{Rank}(Df_a) = 1 \qquad \text{Dim}(\text{Ker } Df_a) + \underbrace{\text{Rank}}_{1} = d+n$