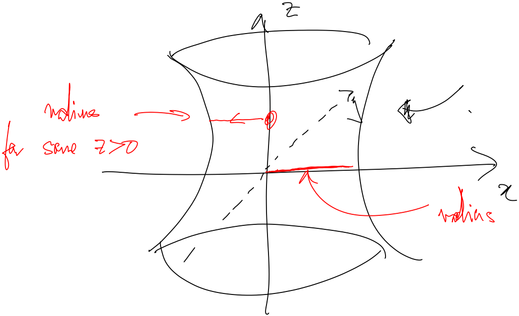
Q4: Surface $z^2 = x^2 + y^2 - 1 \leftarrow S$

Find 2 lines contained on S at (a, b, c).



① fix z. slice:

$x^2+y^2 = 1+z^2$.

Circle of radius $\sqrt{1+z^2}$

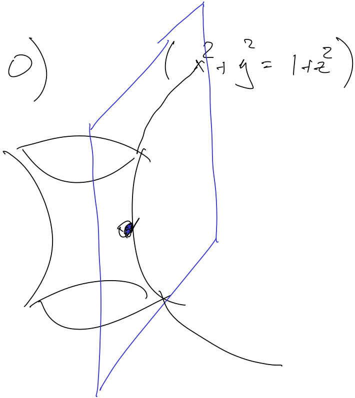
Idea: Pick $(a,b,c) = (1,0,0)$

Compute tgt plane of S at (a,b,c).

$\longrightarrow$ $S = \{f = 0\}$

$f(x,y,z) = x^2 + y^2 - z^2 - 1$

Tgt ~~plane~~ space : $\ker\left(Df_{(a,b,c)}\right)$



$f = x^2 + y^2 - z^2 - 1$ $\qquad Df = \begin{pmatrix} 2x & 2y & -2z \end{pmatrix}$

at (a,b,c) : $Df = \begin{pmatrix} 2a & 2b & -2c \end{pmatrix}$

$\operatorname{Ker}\begin{pmatrix} \cancel{2}a & \cancel{2}b & -2c \end{pmatrix}$ = Tgt space at (a,b,c)

$\hookrightarrow ax + by = cz$

Tangent plane : $\boxed{a(x-a) + b(y-b) = c(z-c)}$

Hair

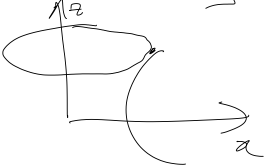
Expect: straight line passing through (a,b,c) contained in S

is also contained in the tgt plane.

$$S \longrightarrow x^2+y^2=1+z^2.$$

$$\text{Tgt plane} \longrightarrow a(x-a)+b(y-b)=c(z-c)$$

Intersection: Solve simultaneously.



$$\text{Tgt plane at } (1,0,0):\ 1(x-1)+0.y=0.z$$

$x^2 + y^2 = 1 + z^2$ & $x = 1$

$\iff y^2 = z^2$ & $x = 1$ $\iff \boxed{x = 1 \text{ \& } y = \pm z}$

$x = 1 \leftarrow$ plane

$y = -z \leftarrow$ plane

intersection = 2 lines

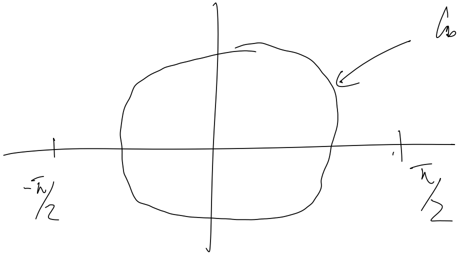
$x = 1 \leftarrow$ plane

$y = +z \leftarrow$ plane

intersection = 1 line

$\boxed{Q1\,f}$

$$\cos x \cos y = \frac{1}{2} \qquad x, y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$



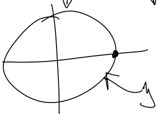
$\cos x \cos y = \frac{1}{2}$

Q: parametrize it.

Simpler Eg: $x^2+y^2=1$ ← parametrize

$x = \cos t$ & $y = \sin t$ | $\gamma(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$ $t \in (0, 2\pi)$

$y = +\sqrt{1-x^2}$



$y = -\sqrt{1-x^2}$

Solve & write x in terms of y (or y in terms of x)

$y = \sqrt{1-x^2}$ or $y = -\sqrt{1-x^2}$

Parametrize: Top half: $\gamma(t) = \begin{pmatrix} t \\ \sqrt{1-t^2} \end{pmatrix}$, bottom half: $\gamma(t) = \begin{pmatrix} t-2 \\ -\sqrt{1-(t-2)^2} \end{pmatrix}$

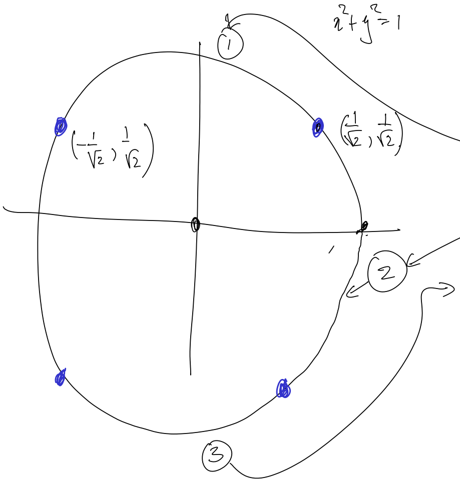
$t \in (-1, 1)$ $t \in (1, 3)$

$$\gamma(t) = \begin{cases} \begin{pmatrix} t \\ \sqrt{1-t^2} \end{pmatrix} & t \in (-1, 1) \\ \begin{pmatrix} 2-t \\ -\sqrt{1-(2-t)^2} \end{pmatrix} & t \in (1, 3) \end{cases}$$

① Is γ cts? (Now yes).

② Is γ C^1? No! not diff at $x = +1$

③ How can you fix this?



Break the param into 4 parts.

1. Top: Use $y = \sqrt{1-x^2}$
2. Right Use $x = +\sqrt{1-y^2}$
3. Bottom Use $y = -\sqrt{1-x^2}$
4. Left use $x = -\sqrt{1-y^2}$

Formula for γ: $\gamma(t) = \begin{cases} \begin{pmatrix} t \\ \sqrt{1-t^2} \end{pmatrix} & t \in \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \text{ (top quarter)} \\ \begin{pmatrix} \sqrt{1-\left(\frac{2}{\sqrt{2}}-t\right)^2} \\ \frac{2}{\sqrt{2}} - t \end{pmatrix} & t \in \left(\frac{1}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right) \text{ (right quarter)} \\ \begin{pmatrix} \end{pmatrix} & t \in \left(\frac{3}{\sqrt{2}}, \frac{5}{\sqrt{2}}\right) \text{ (bottom)} \\ \begin{pmatrix} \end{pmatrix} & t \in \left(\frac{5}{\sqrt{2}}, \frac{2}{\sqrt{2}}\right) \text{ (left).} \end{cases}$

$t = \frac{1}{\sqrt{2}}$ need $y = \frac{1}{\sqrt{2}}$

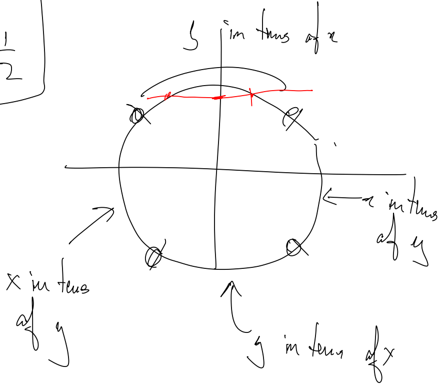
$t = \frac{3}{\sqrt{2}}$ need $y = -\frac{1}{\sqrt{2}}$

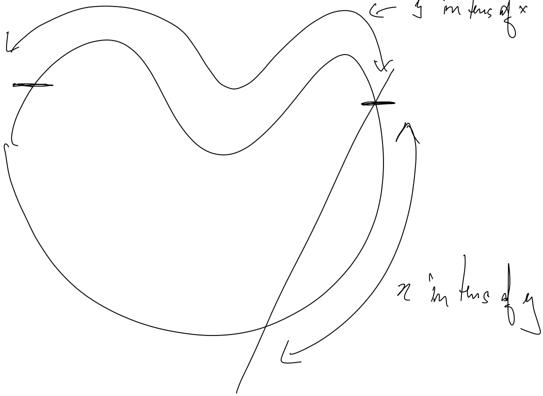
Homework problem: $\boxed{\cos x \cos y = \frac{1}{2}}$

$$\cos y = \frac{1}{2\cos x}$$

$$y = \cos^{-1}\left(\frac{1}{2\cos x}\right) \text{ or (top branch)}$$

$$\text{or } y = -\cos^{-1}\left(\frac{1}{2\cos x}\right) \quad \text{(bot branch)}$$




y in terms of x
x in terms of y