① Update in Grading policy :

① Drop 5 lowest HW.

② ⓐ Better midterm : 30% , Final 50%

ⓑ Each midterm : 20% , Final 40%

better. ⓒ Final 80%.

Midterm 2 : "in class" (at Home!) open book, open notes

Scanning : Android : Adobe Scan
~~iP~~ iPhone : Notes app / Cam Scanner
(Scanner Pro $4).

Math: Lagrange multipliers (Constrained optimization).

Need to know first (1) Manifolds:

(a) 1 dim manifold $\rightarrow$ curves

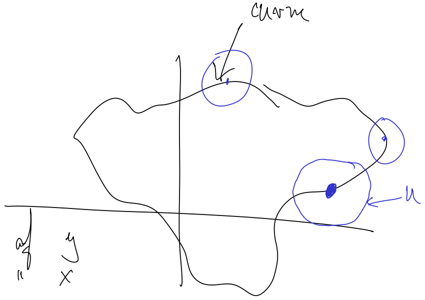
Curve in $\mathbb{R}^2$:

$\Gamma \subseteq \mathbb{R}^2$ is a C^1 curve if

$\forall a \in \Gamma$, $\exists$ an open set $U \ni$

on $\Gamma \cap U$, either x is a a C^1 fn of y

or y " " " " " x



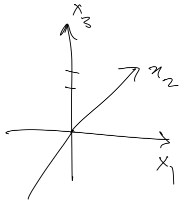
(b) Curve in $\mathbb{R}^3$: $\Gamma \subseteq \mathbb{R}^3$ (x_1, x_2, x_3).

$\forall a \in \Gamma$, $\exists\, U$ s.t. on $\Gamma \cap U$ either

(1) $x_1 \& x_2$ are C^1 fns of x_3

OR (2) x_1, x_3 " " " x_2

OR (3) x_2, x_3 " " " " x_1



(b) We say $\Gamma \subseteq \mathbb{R}^n$ is a surface if

$\forall a \in \Gamma$, $\exists\, U$ open such that on $U \cap \Gamma$

$\exists$ 2 coordinates, $x_i \& x_j$ s.t. on $U \cap \Gamma$ all other coordinates are C^1 fns of $x_i \& x_j$ (i.e. $x_3, \dots x_n$ are C^1 fns of $x_1 \& x_2$

or $x_2, x_4, \dots x_n$ " " " x_1, x_3

etc.).

(c) $M \subseteq \mathbb{R}^n$ is a d-dimensional manifold if
$\forall a \in M$, $\exists U \subseteq \mathbb{R}^n$ open & d-coordinates
$x_{i_1}, x_{i_2}, \ldots x_{i_d}$ $\ni$ on $U \cap M$ the remaining $n-d$
coordinates are C^1 fns of $x_{i_1}, \ldots x_{i_d}$.

Prime examples: $f : \mathbb{R}^{n+d} \longrightarrow \mathbb{R}^n$ is C^1.
$c \in \mathbb{R}^n$.
Let $M = \{f = c\}$. Assume $\forall x \in M$, $\mathrm{Rank}(Df_x) = n$
Then by the implicit fn theorem M is a d-dim manifold.

Need: If $a \in M$, $\boxed{\text{Then Tgt \underline{space} of } M \text{ at } a \text{ is } \operatorname{Ker}(Df_a)}$

Lagrange multipliers: Maximise a fn f given the constraint $g = 0$

$f : \mathbb{R}^d \to \mathbb{R}$, C^1

$g : \mathbb{R}^d \to \mathbb{R}^n$. $\boxed{\{g = 0\}}$ is (typically) $d-n$ dim manifold.

Condition: at a local constrained max/min.
$$\exists \lambda_1, \dots \lambda_n \text{ s.t. } \nabla f(a) = \sum_{i=1}^{n} \lambda_i \nabla g_i(a)$$