21-269 Vector Analysis: Midterm 1.

2019-02-14

- *This is a closed book test. No electronic devices may be used. You may not give or receive assistance.*
- *You have 50 minutes. The exam has a total of 4 questions and* 40 *points.*
- *You may use any result proved in class or any regular homework problem* ***PROVIDED*** *it is* independent *of the problem you want to use the result in. (You must also* ***CLEARLY*** *state the result you are using.)*
- *The questions are all roughly comparable in difficulty, with the first few being slightly easier, and the last few being slightly harder. Good luck* $\ddot\smile$.

10 1. Let (a_n) be a bounded sequence in $\mathbb{R}$. Let $\alpha_n = \inf\{a_k \mid k \geqslant n\}$. Must the sequence (α_n) be convergent? Prove your answer.

10 2. Let $U = \{x \in \mathbb{R}^2 \mid x \neq 0\}$, and suppose $f\colon U \to (0,\infty)$ is a continuous function such that $f(tx) = t^3 f(x)$ for all $t \in (0,\infty)$ and $x \in U$. Define

$$\alpha = \inf\left\{ \frac{f(x)}{|x|^3} \;\middle|\; x \in U \right\}.$$

Must $\alpha > 0$? Prove your answer.

10 3. Prove the intermediate value theorem. Namely, if $f\colon [a,b] \to \mathbb{R}$ is continuous, $f(a) < 0$ and $f(b) > 0$, show that there exists $c \in (a,b)$ such that $f(c) = 0$.

This was a problem on your homework. Please provide a complete proof here, and don't simply quote the homework. You may, however, use any other result proved in class or any regular homework problem **PROVIDED** it is *independent* of this problem and you state the result clearly.

10 4. Define $f\colon \mathbb{R}^2 \to \mathbb{R}$ by $f(x) = \dfrac{x_1 x_2}{|x|}$ when $x \neq 0$, and $f(0,0) = 0$. Is f differentiable at 0? Prove your answer.

NOTE: Here we used the convention that $x = (x_1, x_2) \in \mathbb{R}^2$.