21-268 Multidimensional Calculus: Midterm 2.

2018-03-28

- *This is a closed book test. No electronic devices may be used. You may not give or receive assistance.*
- *You have 50 minutes. The exam has a total of 5 questions and 40 points.*
- *You may use any result proved in class or any regular homework problem **PROVIDED** it is* independent *of the problem you want to use the result in. (You must also **CLEARLY** state the result you are using.)*
- *The questions are roughly in increasing order of difficulty. Good luck ◡̈.*

$\boxed{6}$ 1. Consider the system of equations

$$x^3y + y^3z + z^3x = 0\,, \qquad \text{and} \qquad x + y + z + e^{xyz} = 2\,.$$

to which $(1, 0, 0)$ is a solution. Near this point, which of the following statements is guaranteed by the implicit function theorem.

(a) x and y can be expressed as differentiable functions of z.

(b) x and z can be expressed as differentiable functions of y.

(c) y and z can be expressed as differentiable functions of x.

No justification is required. Incorrect answers are worth no credit, blank answers are worth 25% credit, and correct answers are worth full credit.

$\boxed{5}$ 2. Let $\Gamma = \{x \in \mathbb{R}^3 \mid 2x_1 + 6x_2^2 + 8x_3^3 = 16\}$. Determine whether Γ is a curve or surface. If Γ is a curve, find an equation for the tangent line at the point $(1, 1, 1)$. If Γ is a surface, find an equation for the tangent plane at the point $(1, 1, 1)$.

$\boxed{9}$ 3. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be a C^2 function, and define $g\colon \mathbb{R}^2 \to \mathbb{R}$ by

$$g(u, v) = f(u^2 - v^2, 2uv)\,.$$

Compute $\partial_u \partial_v g$, and express your answer explicitly in terms of u and v and derivatives of the function f.

4. Let $f, g\colon \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = x$ and $g(x, y) = y^2 - x^3 + 2x$.

$\boxed{7}$ (a) Using Lagrange multipliers, find all points at which the function f can attain a constrained local minimum or maximum, subject to the constraint $g = 0$.

$\boxed{3}$ (b) Determine whether each of the points you found in the previous part is a constrained local minimum, maximum or neither. No proof or justification is required for this part. Incorrect answers are worth no credit, blank answer are worth 25% credit, and a correct answer is worth 100% credit.

$\boxed{10}$ 5. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be a C^2 function, and $a \in \mathbb{R}^2$. Suppose $\nabla f(a) = 0$ and for all $v \in \mathbb{R}^2$ we have $(Hf_a v) \cdot v \geqslant 10|v|^2$. Show directly f attains a local minimum at a.

NOTE: We proved in class that if $\nabla f(a) = 0$ and Hf_a is positive definite, then f attains a local minimum at a. You may **NOT** use this result here as this problem is a special case of the result we proved in class. Provide a direct proof of this problem. You may use other (independent) results from class or the homework.